

Ques No. 1:-

A. State and prove SAS theorem of similarity of two triangles (4)

B. Answer any two of the following (5)

(1) Simplify

$$\frac{1}{1+x^2-y^2} + \frac{x^2}{x^2xy-x+y} - \frac{y^2}{xy-x+y-y^2}$$

(2) Factorize

$$a^2b^2(a-b) + b^2c^2(b-c) + c^2a^2(c-a)$$

(3) Factorize

$$2ab(a+2b) + 2bc(b+2c) + 2ca(c+2a) + 9abc$$

C. Factorize any two of the following: (4)

(1) $9x^3 + 6x^2 - 5x - 2$ Standard

(2) $(x+2)(x+3)(x+6)(x+9) - 35x^2$

(3) $(x^2-4)(y^2-9) - 24xy$

D. Solve any one

(1) Find the reduced form of $\left(\frac{x+1}{x-1} + \frac{x-1}{x+1}\right) \times \frac{x^2-1}{x^2+1}$ (2)

(2) If $\frac{x^3+27x}{9x^2+27} = \frac{172}{171}$ then find x.

E. Fill in the blanks (4)

(1) If $f(x) = 2x^0 + (x-1)^{\frac{1}{2}}$ then $f(5) = \dots$ (0.5, 2.5, 10.5)

(2) If $f: \mathbb{R}^+ \rightarrow \mathbb{R}$, $f(x) = \log_3 x$ then $f(81) = \dots$
(3, 4, 9)

(3) Control unit is a part of ... (CPU, IPD, OPD)

(4) In flow chart the symbol for input is
----- (◇, □, □)

A - Prove that in the congruent circles, chords equidistant from the centres are congruent (4)

B - Solve any two (6)

(1) If $\frac{a^2+a+1}{b^2+b+1} = \frac{a^2-a+1}{b-b+1}$ then prove that

either a and b are equal or both of them are multiplicative inverse.

(2) If a, b, c, d are in proportion then prove that $\frac{a^3+b^3+c^3+d^3}{a^{-3}+b^{-3}+c^{-3}+d^{-3}} = (ad)^3$

(3) The expenses of a tour is partly constant and partly varies directly as the number of students. When the number of students are 40 and 25 the expense per student are Rs 75 and Rs 90 respectively. If number of students are 50 then find the expenditure per student.

C - Solve any two (4)

(1) The mean of data is three-fourth part of the median. If mode of the data is 50 find the mean.

(2) For a data $\bar{x} - M = 2$ and $\bar{x} + M = 20$ find the mode.

(3) The median of data $\frac{x}{5}, \frac{x}{4}, x, \frac{x}{2}, \frac{x}{3}$ where $x > 0$ is 8. Find mean.

D - Solve any one (2)

(1) If $f: \mathbb{Z} \rightarrow \mathbb{Z}, f(x) = x^2 - x - 2$ find $\frac{1}{x} [f(x+1) - f(x)]$

(2) $f: A \rightarrow \mathbb{N}, f(x) = \log_2 x$, if $R_f = \{1, 2, 3, 4\}$ then find A .

E. Fill in the blanks

(1) $\frac{(2a-2)^3}{8(1-a)^3} + \frac{(2x-2)^4}{8(1-x)^4} + \frac{(2-2x)^5}{16(x-1)^5} = \dots$ $(-1, 0, 1)$ (4)

(2) If $\frac{2a}{5} = \frac{3b}{2} = \frac{6c}{5} = \frac{a+b+c}{m}$ then $m = \dots$ $(-4, 3, 4)$

(3) $x + \frac{1}{y}$ & $x - \frac{1}{y}$ then $x^2 = \dots$ $(x, \frac{1}{y}, y^2)$

(4) If $\sin 2\theta = \cos 3\theta$ then $\theta = \dots$ $(0, 18, 90)$

Ques NO. 3 -

A. Prove that an angle inscribed in a semi circle is a right angle. (4)

B. Solve any two: (6)

(1) From a point at a height of 3.75 m above the surface of a lake, the angles of elevation and depression of the top of a temple and its reflected image in the lake are 30° and 60° respectively. Find the height of the temple above the lake.

(2) A regular hexagon of side 6 cm is cut out from a plane circular metal sheet of radius 6 cm. Find the area of the remaining portion of the sheet. ($\sqrt{3} = 1.73$)

(3) The diameter of a metallic sphere is 3 cm. It is melted and drawn into a wire of diameter 2 mm. Find the length of wire.

C. Solve any Two (4)

(1) Prove that $\sin^6 \theta + \cos^6 \theta = 1 - 3 \sin^2 \theta \cos^2 \theta$

(2) Evaluate: $2 \cos^3 60 - 12 \sin^4 60 + \frac{3}{4} \tan^2 30 + 12 \cot 45$

(3) If $\cos \theta + \cos^2 \theta = 1$ then prove that $\sin^2 \theta + \sin^4 \theta = 1$

D- Solve any one

(1) Prove $(3 \sin \theta + 5 \cos \theta)^2 + (3 \cos \theta - 5 \sin \theta)^2 = 34$

Let $x \propto \frac{1 + \cos \theta}{\sin \theta}$ and $y \propto \frac{1 - \cos \theta}{\sin \theta}$ then

Prove that $x \propto \frac{1}{x}$

E- Answer the following

(1) Define Median of a Triangle

(2) Define Altitude of a triangle

(3) The difference of two roots of $ax^2 + bx + c = 0$ is 3. Find its discriminant.

(4) Find the quadratic equation whose roots are $\frac{2}{3}$ and $\frac{3}{2}$.

Ques NO. 4 -

A- In $\triangle ABC$, $\angle B$ is a right angle and BM is the altitude on the hypotenuse. Prove that

$$\frac{1}{BM^2} = \frac{1}{AB^2} + \frac{1}{BC^2}$$

B- Solve any two

(1) The mean of the following frequency distribution is 3.42 for 50 frequency.

x	1	2	3	4	5	6
f	a	7	9	12	10	b

find a and b.

(2) Find mean

class	20-29	30-39	40-49	50-59	60-69	70-79
f	15	16	38	15	9	7

(3) Simplify: $\left(\frac{1}{6x^2 + 17x + 12} + \frac{1}{12x^2 + 31x + 20} \right) \div \frac{1}{8x^2 + 22x + 15}$

C- Solve any two

(1) The distance between the two chords 10 and 24 are on ~~opposite~~ ^{same} sides and parallel to a diameter. If their distance is 7 then find the diameter of the circle.

(2) For the arc PQ of $\odot (O, 4)$, $m\angle POQ = 45^\circ$ find the length of major arc PQ.

(3) Three circles of centres P, Q and R touch to each other externally. If $PQ = 4$, $QR = 6$ and $PR = 8$ find the sum of their radii.

D- Solve any one (2)

(1) If $x^2 + (k+4)x + 4(x+4) = 0$ has two equal roots then find the values of k.

(2) Find the roots of $x^2 + (x+1)^2 = 1861$

E- Answer the following: (4)

(1) Define cyclic quadrilateral

(2) All the points of a chord of a circle are interior of the circle. Comment.

(3) Define: Arc of a circle.

(4) The volume of a sphere is 4851 find its ^{diameter.} radius.

Ques NO. 5-

A- If $PO = 10$ then construct $\triangle ABC$ such that the length of hypotenuse $BC = PO$ and $AB = \frac{1}{2} PO$. Write the all steps of constructions. (4)

B- Solve any two: (6)

(1) If $2x + 3y < 3x + 2y$ then prove that $x^3 + y^3 < (x+y)^3$

(2) Find the roots of $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

(3) The perimeter of a right angled triangle is 56 and the sides containing the right angle differ by 17. Find the length of each sides of the triangle.

C- Solve any two: (4)

(1) In $\triangle ABC$, \overrightarrow{AD} bisector of $\angle A$. $B-D-C$. If $AB = \frac{2}{3} AC$ and $BC = 12.5$ then find BD .

(2) In triangle ABC , $D \in \overline{AB}$, $CD = 6$, $BD = 9$, $BC = 12$ and $\angle CAB = \angle BCD$. find the perimeter of $\triangle ADC$.

(3) The base of an isosceles triangle is 24 and

Perimeter is 98. Find the length of altitude on the base of triangle.

D. Solve any one:

(2)

(1) Find the volume of an atom whose diameter is 10^{-8} ($\pi = 3.14$)

(2) The curved surface area of a cone is 550 cm^2 and its slant height is 25 cm. Find its volume.

E. For cyclic quadrilateral ABCD, \vec{AE} is a bisector of $\angle A$ and \vec{CE} is a bisector of $\angle C$. Prove that \vec{AC} is a diameter of the circumcircle of ABCD. :OR:

(4)

F. Answer the following question.

(1) Chord $XY = 19.2$ of $\odot(O, 10)$ find the distance of the chord from the centre.

(2) In $\odot(O, r)$ a chord QR is such that $m\angle QPR = 120$ and $QR = 4\sqrt{3}$ find r . centre P and O

(3) Two coplanar circles of radii 7.5 and 13.5 touch to each other. Find the possible values of PQ .

(4) $\angle ABC$ is an angle inscribed in a semicircle. If $\triangle ABC$ is an isosceles triangle and $AB = 8$ then find the radius of the circle.



Ques NO. 1 -

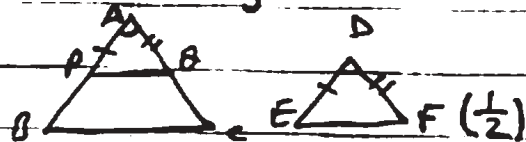
LESS: 1 1

A- Statement: If for a correspondence between two triangles, two pairs of corresponding sides are proportional and the included angles are congruent, then the correspondence is a similarity (1/2)

Given: In $\triangle ABC$ & $\triangle DEF$, $\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle A \cong \angle D$

To prove that $ABC \leftrightarrow DEF$ is similarity

Proof:



There can exist two relations

(1) If $AB = DE$:

If $AB = DE$ then $\frac{AB}{DE} = \frac{AC}{DF} = 1$ or $AC = DF$ (1/2)

$\therefore \triangle ABC \cong \triangle DEF$ (By SAS Postulate)

Hence $\angle B \cong \angle E$ and $\angle C \cong \angle F$

$\therefore ABC \leftrightarrow DEF$ is a similarity (1/2)

(2) If $AB \neq DE$ then $AB > DE$ or $AB < DE$

Let $AB > DE$ Hence $\frac{AB}{DE} > 1$

By point plotting theorem construct $AP = DE$ & $AD = DF$ (1/2)

$\therefore \triangle APD \cong \triangle DEF$ (SAS Postulate) (1/2)

$\therefore \angle APD \cong \angle E$ and $\overrightarrow{PD} \parallel \overrightarrow{BC}$

Hence $ABC \leftrightarrow DEF$ is similarity (1/2)

Similarly for $AB < DE$ we can prove that

$ABC \leftrightarrow DEF$ is a similarity. (1/2)

B- (1)
$$\frac{1}{(x-1)(y-1)} + \frac{x^2}{(x-y)(x-1)} - \frac{y^2}{(x-y)(y-1)}$$
 (1)

$$\frac{(x-y)(1+xy-x-y)}{(x-y)(1-x)(1-y)}$$
 (1)

$$\therefore \frac{(x-y)(1-x)(1-y)}{(x-y)(1-x)(1-y)} = 1$$
 (1)

(2)
$$a^3b^2 - a^2b^3 + b^3c^2 - b^2c^3 + a^2c^3 - a^3c^2$$

$$(b-c)(a^3b + a^3c - a^2b^2 - a^2bc - a^2c^2 + b^2c^2)$$
 (1)

$$(a-b)(b-c)(a^2b + a^2c - ac^2 - bc^2)$$
 (1)

$$+ (a-b)(b-c)(c-a)(ab+bc+ca)$$
 (1)

(3) $2a^2b + 4a^2c + 4ab^2 + 9abc + 2ac^2 + 2b^2c + 4bc^2$ (1)

$2a^2(b+2c) + a(b+2c)(4b+c) + 2bc(b+2c)$

$(b+2c)(2a^2 + 4ab + ac + 2bc)$ (1)

$(a+2b)(b+2c)(c+2a)$ (1)

C- (1) $(x+1)(9x^2 - 3x - 2)$ (1)

$(x+1)(3x-2)(3x+1)$ (1)

(2) $(x^2 + 10x + 18)(x^2 + 9x + 18) - 35x^2$

$m^2 + 20mx + 64x^2$ (Let $x^2 + \alpha = m$) (1)

$(m+16x)(m+4x)$

$(x^2 + 16x + 18)(x^2 + 4x + 18)$ (1)

(3) $x^2y^2 - 12xy + 36 - 9x^2 - 12xy - 4y^2$ (1)

$(xy-6)^2 - (3x+2y)^2$

$(xy+3x-2y-6)(xy-3x-2y-6)$ (1)

D (1) $\frac{(x+1)^2 + (x-1)^2}{(x-1)(x+1)} \cdot \frac{x^2+1}{x^2+1}$ (1)

$= \frac{2(x^2+1)}{(x-1)(x+1)} \cdot \frac{(x+1)(x-1)}{x^2+1}$

$= 2$ (1)

(2) $\frac{x^3 + 27x + 9x^2 + 27}{x^3 + 27x - 9x^2 - 27} = \frac{172+178}{172-171}$ (Cantor & Dividends) (2)

$\frac{x+3}{x-3} = \frac{7}{1}$ (2)

$\frac{x+3+x-3}{x+3-x+3} = \frac{7+1}{7-1}$ (2)

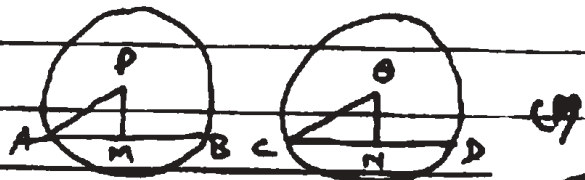
$x=4$ (2)

E- (1) 2.5 (2) 4 (3) CPU (4) (1 mark for each = 4)

Ques NO. 2 -

A- Given: \overline{AB} and \overline{CD} are chords respectively in congruent circles with centre P and O. $\overline{PM} \perp \overline{AB}$ and $\overline{ON} \perp \overline{CD}$, where $\overline{PM} \cong \overline{ON}$.

To Prove that: $\overline{AB} \cong \overline{CD}$



Proof: $\overline{PM} \perp \overline{AB}$ and M is foot of perpendicular. Date: / /

$\therefore AB = 2AM$ --- (1) (by theo. 12) (1)

Similarly $CD = 2CN$.

$\triangle AMN \cong \triangle CNM$ (by R.H.S theorem) (1)

$\therefore AM = CN$ or $AB = CD$

then $\overline{AB} \cong \overline{CD}$. (1)

B- (1) $\frac{a^2+a+1}{a^2-a+1} = \frac{b^2+b+1}{b^2-b+1}$ (by Alternando) (1/2)

$\frac{a^2+1}{a} = \frac{b^2+1}{b}$ (by Compo. & Dividendo) (1)

$a^2b + b = ab^2 + a$ (1/2)

$(a-b)(ab-1) = 0$ (1/2)

$\therefore a = b$ or $a = \frac{1}{b}$ (1/2)

(2) let $\frac{a}{b} = \frac{c}{d} = k$

$\therefore a = bk, c = dk$. (1/2)

$\frac{b^3k^3 + b^3 + a^3k^3 + d^3}{b^3k^{-3} + b^{-3} + d^3k^{-3} + d^{-3}}$ (1/2)

$\frac{(b^3+d^3)(k^3+1)}{(\frac{1}{b^3} + \frac{1}{d^3})(\frac{1}{k^3} + 1)}$ (1)

$\frac{(b^3+d^3)(k^3+1) b^3 d^3 k^3}{(b^3+d^3)(k^3+1)}$ (1/2)

$\frac{(a \cdot d)^3}{(a \cdot d)^3}$ (1/2)

(3) $y = ax + b$ (let $y = \text{Expense}$, $x = \text{no. of student}$)

$40 \times 75 = 40a + b$ and $25 \times 90 = 25a + b$ (1)

By solving $a = 50$ and $b = 1000$

$\therefore y = 50 \times 50 + 1000 = 3500$ (1)

$\therefore \text{Expense per student} = 70 \text{ Rs.}$ (1)

C (1) $\bar{x} = \frac{3}{4}M$, $\therefore 3M = 4\bar{x}$. (1/2)

$Z = 3M - 2\bar{x}$ (1/2)

$50 = 4\bar{x} - 2\bar{x}$, $\bar{x} = 25$ (1)

(2) By solving $\bar{x} = 11$, $M = 9$ (1)

But $Z = 3M - 2\bar{x} = 27 - 22 = 5$ (1)

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(3) $\frac{x}{2}, \frac{x}{4}, \frac{x}{3}, \frac{x}{2}, x$

\therefore Median = $\frac{x}{2} = 8, x = 24$ (1/2)

\therefore data = 4, 8, 6, 8, 12, 24 (1/2)

\therefore Mean = $\frac{4+8+6+8+12+24}{5} = \frac{54+8}{5} = 10.96$ (1)

D- (1) $f(x+1) = x^2 + x - 2$ (1/2)

$f(x+1) - f(x) = 2x$ (1)

$\frac{1}{2} [f(x+1) - f(x)] = 2$ (1/2)

(2) $f(x) = 4x, x = 1, \therefore x = 2$ (1/2)

Similarly $x = 4, x = 8, x = 16$ (1)

$A = \{2, 4, 8, 16\}$ (1/2)

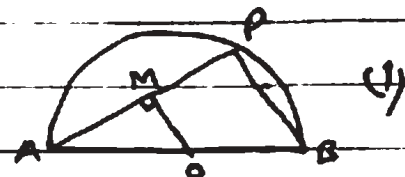
E- (1) -1 (2) 4 (3) $\frac{1}{y}$ (4) 18 (1 mark for each = 4)

Ques NO. 3 -

A- Given: \widehat{AB} is semicircle, O centre, $\angle APB$ is an angle inscribed in the semicircle.

To prove: $m\angle APB = 90$

Pf: \overline{AP} is chord, $\overline{OM} \perp \overline{AP}$



\therefore O and M are mid points

$\therefore \overline{OM} \parallel \overline{PB}$ and \overline{PA} is transversal.

$\therefore \angle AMO \cong \angle APB$ (corresponding angles) (1)

$\therefore m\angle APB = m\angle AMO = 90$

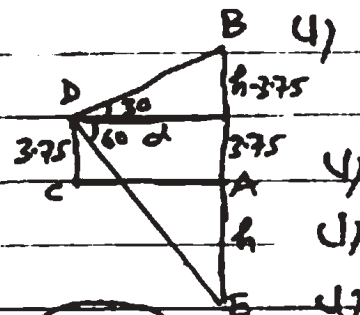
$\therefore \angle APB$ is a right angle.

B- (1) Let height of temple $AB = h$

$\therefore \tan 30 = \frac{h-3.75}{d}$ or $d = \sqrt{3}(h-3.75)$

$\tan 60 = \frac{h+3.75}{d}$ or $d = \frac{h+3.75}{\sqrt{3}}$

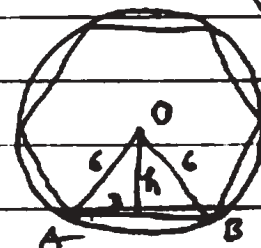
By equating d we get $h = 7.5$



(2) Area of triangle = $\frac{1}{2} AB \times h$

$h^2 = 6^2 - 3^2$ or $h = 3\sqrt{3}$

\therefore Area of triangle = $\frac{1}{2} \times 6 \times 3\sqrt{3} = 9\sqrt{3}$



Area of minor segment

= Area of sector - Area of $\Delta = \frac{\pi r^2 \theta}{360} - 9\sqrt{3} = \frac{132}{7} - 9\sqrt{3} = 3.28$ (1)

\therefore Remaining Area = $6 \times 3.28 = 19.68$ Sq. Cm. (1)

(3) volume of sphere = $\frac{4}{3}\pi r^3 = \frac{9}{2}\pi$ (1)

volume of wire = $\pi r^2 h = \frac{\pi h}{100}$ (1)

$\therefore \frac{\pi h}{100} = \frac{9\pi}{2}$, $h = 450$ cm (1)

C-(1) LHS: $(\sin^2\theta + \cos^2\theta)(\sin^4\theta - \sin^2\cos^2\theta + \cos^4\theta)$ (1)

= $1[(\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta]$ (1)

= $1 - 3\sin^2\theta\cos^2\theta$ (1)

(2) = $2\left(\frac{1}{2}\right)^2 - 12\left(\frac{\sqrt{3}}{2}\right)^4 + \frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2 + 12(1)$ (1)

= $\frac{1}{2} - 27 + \frac{1}{2} + 12$ (1)

= $\frac{23}{4}$ (1)

(3) $1 - \cos^2\theta = \cos\theta$ or $\sin^2\theta = \cos\theta$ (from given) (1)

LHS.

= $\cos\theta + \cos^2\theta$ (1)

= 1

D-(1) LHS: $9\sin^2\theta + 30\sin\theta\cos\theta + 25\cos^2\theta + 9\cos^2\theta - 3\sin\theta\cos\theta + \frac{2}{25\sin\theta}$ (1)

= $34\sin^2\theta + 34\cos^2\theta$ (1)

= 34 (1)

(2) $x = k\left(\frac{1+\cos\theta}{\sin\theta}\right)$ and $y = m\left(\frac{1-\cos\theta}{\sin\theta}\right)$ where $k, m \neq 0$ (1)

$xy = km$ ($km \neq 0$) (1)

$\therefore x \propto \frac{1}{y}$ (1)

E-(1) Median - A line segment whose end points lie on a vertex and the mid point of the triangle is called median. (1)

(2) Altitude - A perpendicular line-segment is drawn from the vertex to the line containing opposite side of a triangle is called altitude. (1)

(3) $\alpha - \beta = 3$, $\frac{-b+\sqrt{\Delta}}{2a} - \frac{-b-\sqrt{\Delta}}{2a} = 3$ (1)

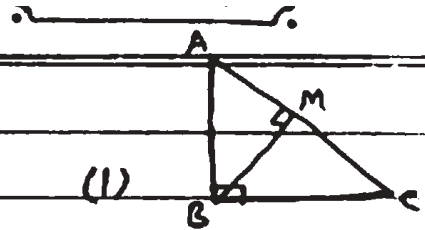
$\therefore \sqrt{\Delta} = 3a$ or $\Delta = 9a^2$ (1)

(4) $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ (1)

$6x^2 - 13x + 6 = 0$ (1)

Ques 4 -

A- Given, $\triangle ABC, \angle B = 90, \overline{BM} \perp \overline{AC}$



To prove that $\frac{1}{BM^2} = \frac{1}{AB^2} + \frac{1}{BC^2}$ (1)

Proof: From geometric mean theorem

We know $AB^2 = AM \times AC, BC^2 = MC \times AC, BM^2 = AM \times MC$ (1)

$$\therefore \frac{1}{AB^2} + \frac{1}{BC^2} = \frac{1}{AM \times AC} + \frac{1}{MC \times AC} = \frac{1}{AM \times MC}$$
 (1)

$$= \frac{1}{BM^2} \quad \therefore \frac{1}{BM^2} = \frac{1}{AB^2} + \frac{1}{BC^2}$$
 (1)

$$\therefore \frac{1}{BM^2} = \frac{1}{AB^2} + \frac{1}{BC^2}$$
 (1)

B (1) $\bar{x} = \frac{\sum fx_i}{N} = \frac{a + 14 + 27 + 48 + 50 + 6b}{N}$ (1)

$$\therefore 3.42 = \frac{a + 6b}{50} \quad \text{or } a + 6b = 32 \quad (1)$$

$$\text{But } a + b = 50 - (7 + 9 + 12 + 10) = 12 \quad (2)$$

from (1) & (2) $a = 8$ and $b = 4$ (1)

(2) $\bar{x} = A + \frac{\sum f_i u_i}{N} \times C$ $\sum f_i u_i = 8$ (1)

$$= 44.5 + \frac{8}{100} \times 10$$
 (2)

$$= 45.3$$
 (1)

(3) $\left[\frac{1}{(2x+3)(3x+4)} + \frac{1}{(4x+5)(3x+4)} \right] \times \frac{(2x+3)(4x+5)}{1}$ (1)

$$= \frac{6x+8}{(2x+3)(3x+4)(4x+5)} \times \frac{(2x+3)(4x+5)}{1}$$
 (1)

Ans 2 -

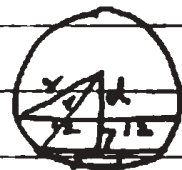
C- (1) $r^2 = d^2 + 12^2$ (1)

and $r^2 = (d+7)^2 + 5^2$ (1)

$$\therefore d^2 + 12^2 = (d+7)^2 + 5^2$$

Hence $d = 5$ (1)

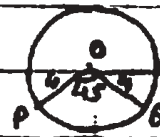
$$\therefore r^2 = 5^2 + 12^2 \quad \text{or } r = 13 \quad \text{or diameter} = 14$$
 (1)



(3) Length of major arc = Circumference - length of

$$= 2\pi r - \frac{\pi r \theta}{180} \quad \text{minor arc}$$
 (1)

$$= 8\pi - \pi = 7\pi$$
 (1)



(3) $r_1 + r_2 = 4, r_2 + r_3 = 6, r_3 + r_1 = 8$ (1)

By solving $r_1 + r_2 + r_3 = 9$ (1)

D- (1) A quadrilateral, having all its four vertices on the same circle is called a cyclic quadrilateral. (1)

(2) It is wrong statement, because end points of a chord lie on the circle. (1)

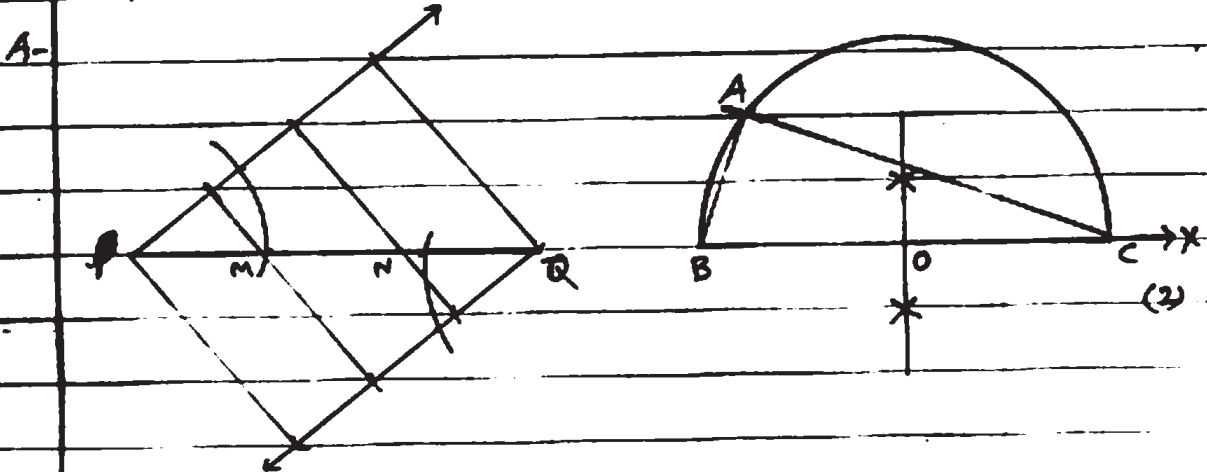
(3) If A and B are any two points on a circle, then the set of all the points of the circle lying in any half closed plane of \overleftrightarrow{AB} is called an arc. (1)

(1) Volume of sphere = $\frac{4}{3}\pi r^3 = 4851$ (1/2)

$\therefore r^3 = \left(\frac{21}{2}\right)^3$ or $r = \frac{21}{2} = 10.5$ (1/2)

\therefore Diameter = $2r = 21$ (1/2)

Ques NO. 5-



Given: $PQ = 10\text{ cm}$

To construct: $\triangle ABC$, $\angle A = 90^\circ$, $BC = PQ$ and $AB = \frac{1}{2}PQ$.

Construction: (1) Trisect $PQ = 10$ in M and N such that $P-M-N-Q$ by construction of dividing a segment in equal parts.

(2) Construct $\odot(B, PM)$ such that it intersects \overrightarrow{BX} in C. (1/2)

(3) Bisect BC in O by the construction of bisector. (1/2)

(4) Draw $\odot(O, OB)$ by compass postulate.

(5) Draw $\odot(B, PM)$ which intersects $\odot(O, OB)$ at A. (1/2)

(6) $\triangle ABC$ is required triangle. (1/2)

B- (1) $2x + 3y = k(3x + 2y)$ ($k \neq 0$)

$x = my$ ($m \neq 0$). (1)

or $\frac{x^2 + y^2}{(x-y)^2} = \text{constant}$. (1/2)

LHS $\frac{m^3y^3+y^3}{(my-y)^3} = \frac{m^3+1}{(m-1)^3} = \text{Constant}$ (1)
 = RHS.

Hence $x^3+y^3 \propto (x-y)^3$ (1/2)

(2) $\Delta = b^2 - 4ac = 32$ (1)

$x = \frac{2\sqrt{2} \pm \sqrt{32}}{2\sqrt{3}}$
 $= \frac{\sqrt{2} \pm 2\sqrt{2}}{\sqrt{3}}$ (1)

$\alpha = \sqrt{\frac{2}{3}}$ and $\beta = -\sqrt{\frac{2}{3}}$ or $\{\sqrt{\frac{2}{3}}, -\sqrt{\frac{2}{3}}\}$ (1)

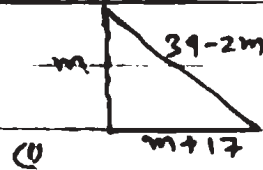
(3) From Perimeter, hypotenuse = $39 - 2m$

$\therefore (39 - 2m)^2 = m^2 + (m + 17)^2$

$\therefore m^2 - 95m + 616 = 0$

$m = 7, m = 88$ (Not Possible)

\therefore Sides are 7, 24, 25.



C- (1) $\frac{AB}{AC} = \frac{BD}{CD}$
 $\frac{2}{3} = \frac{BD}{12.5 - BD}$

$\therefore BD = 5$

(2) $\triangle ABC \sim \triangle CBD$,

$\therefore \frac{AB}{12} = \frac{12}{9} = \frac{AC}{6}$ or $AB = 16$ and $AC = 8$

Perimeter of $\triangle ABC = (16 - 9) + 6 + 8 = 21$

(3) Congruent sides are $\frac{98 - 24}{2} = 37$

$\therefore h^2 + 2^2 = 37^2$

$\therefore h = 35$

D- (1) Volume of sphere = $\frac{4}{3} \pi r^3$ (atom is a sphere/shaft)

$= 5.23 \times 10^{-25}$

(2) Curved surface area of cone = $\pi r l = 550$

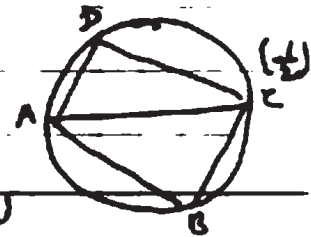
$\therefore r = 7$ and by $l^2 = r^2 + h^2$, $h = 24$

\therefore volume of cone = $\frac{1}{3} \pi r^2 h = 1232$ C.C.

E- Given: $\square ABCD$ is cyclic, \vec{AC} and \vec{BD} are bisectors of $\angle A$ and $\angle C$.

To Prove that: \vec{AC} is diameter

Pf: $\frac{1}{2} m \angle DAB = m \angle DAC$ (1)



And $\frac{1}{2} m \angle DCB = m \angle BCA$ (DATE: / / 2000) (t)

From (1) and (2) and by cyclic quadrilateral properties

$m \angle BAC + m \angle BCA = 90$ (t)

$\therefore \angle ABC = 90$ (t)

Points, A, B, C, D lie on circle (t)

Hence $\angle ABC$ is inscribed in semicircle. (t)

$\therefore \overline{AC}$ is diameter of the circle. (t)

E (1) $d^2 = 10^2 - 9 \cdot 6^2$ (t)

$d = 2.8$ (t)

(2) $r^2 = (2\sqrt{3})^2 + (\frac{r}{2})^2$ (t)

$r = 4$ (t)

(3) $PQ = 13.5 + 7.5 = 21$ (for externally) (t)

$PQ = 13.5 - 7.5 = 6$ (for internally) (t)

(4) $\text{Diameter}^2 = 8^2 + 8^2 = 128$

$\text{Diameter} = 8\sqrt{2}$

$\text{radius} = 4\sqrt{2}$