Subject : MATHEMATICS

Sub Code : 028(E)

**Time : 3 hrs**

**Total Marks : 100**

**Instruction :**

(1) There are FIVE questions in this question paper

(2) Each question carries 20 marks

(3) Figures on right show the total marks of the questions

Q.1

(A) State the converse statement of Pythagores theorem and prove Pythagores theorem. **(4)**

(B) Solve any two of the following **(6)**

(1) Factorize : $a^3 + b^3 + c^3 - 3abc$

(2) Factorize : $a^2(b+c) + b^2(c+a) + c^2(a+b) + 2abc$

(3) Simplify : $\frac{x+1}{x+2} + \frac{x-1}{x-2} + \frac{1}{x+1} + \frac{1}{x-1}$

(C) Factorize: (Any Two) **(4)**

(1) $(x^2 - 9)(y^2 - 25) - 60xy$

(2) $2x^3 + 3x^2 - 3x - 2$

(3) $81x^4 + 4$

(D) Solve any one of the following **(2)**

(1) Simplify : $\frac{x}{x+1} \times \frac{x+1}{x+2} \times \frac{x+2}{x+3} \times \frac{x+3}{x+4} \times \frac{x+4}{x+5} \div \frac{x}{x+5}$

(2) If $\frac{a^2 + b^2 + c^2}{a^2 + ac + c^2} = \frac{b^2}{ac}$

then prove that b is a geometric mean of a and c

(E) Fill in the blanks by selcting the proper alternative from those given in the brackers. **(4)**

(1) If $f : R^+ \cup \{0\} \longrightarrow R^+ \cup \{0\}$, $f(x) = |x|$ i.e. absolute value of x, then graph of f is ____________________________.

(Line segment, Ray, Line parallel to the X-axis)

(2) Inequality set $\{ X \in R / -3 < X < 3 \}$ can be expressed as

( (-3,3), [-3,3], (-3 3] )

(3) In a flow chart, the symbol for output process is

(4) Pictorial representation of the logical plan for getting the solution of problem is called ____________________________

( algorithm, programming, flow-chart)

Q.2

(A) Prove that in perpendicular drawn through the centre of a circle on a chord bisects the chord.

(B) Solve any two of the following

(1) In a $\Delta ABC$, m $\angle A = 90$, $\overline{AD} \perp \overline{BC}$, $D \in \overline{BC}$ then prove that CA is a geometric mean of CD and BC

(2) If $\frac{a^2 - a + 1}{b^2 - b + 1} = \frac{a^2 + a + 1}{b^2 + b + 1}$ ( $a = 0, b = 0, a = b$)

then prove that a is an inverse of b.

(3) The height of a cylinder varies directly as the volume and inversely as the square of the radius of the cylinder when the volume of the cylinder is 125.6 CC, its radius is 2 cm and the height 10 cm. Find the radius of the cylinder whose volume is 9 CC and the height is 8 cm.

(C) Solve any two of the following

(1) Observations of some data are X/5, X, X/4, X/2, and X/3 where X>0. If the median of the data is 8, find the value of X. What wili be the mean of data.

(2) The mean and median of a grouped data are 28.2 and 30.5 respectively Find the mode of the data.

(3) Find the mean of the data with observatios 2/3, 5/3, 1/3, 5/6, 1/6

(D) Solve any one of the following

(1) $f : N \rightarrow Z$ $f(x) = (-1)^x.x$. Find the Rf.

(2) Let A be the set of all acute angles in the plane B = (0,90) and f(x) = (measure of the complementary angle of x, indegree) find Rf.

(E) Fill in the gaps by selecting the proper alternative from those given in the brackets.

(1) $\frac{x^2 + 1}{x^2 - 1} \div \frac{x^2 + 1}{x^2 - 1}$ = ________ ( 1,0,-1)

(2) If a,b,c and d are in proportaion than ________ is true according to the law of invertendo. ( b/a = d/c, b/a = c/d, a/b = c/d )

(3) $2x^2 = 5y$, then $y \propto$ ______ ( $x, x^{2/3}, x^{3/2}$ )

(4) $\frac{2 \sec\theta}{1 + \tan^2\theta}$ = ________ ( $2\sec\theta, 2\cos\theta, 2$)

Q.3

(A) $\overline{PQ}$ is a diameter of a circle. R is a point on a circle other than P and Q. Prove that $\angle PRQ$ is a right angle. (4)

(B) Solve any two (6)

(1) Distance between two pillars of equal heights is 200 mtrs. The angle of elevation of their tops from some point on the line segment joining their base are $60^0$ and $30^0$ respectively. Find the height of the pillar.

(2) In a circle of radius 10 cm. $\overline{OA}$ and $\overline{OB}$ are two mutually perpendiculars radius. Find the areas of the minor segment associated with $\angle AOB$ ( $\pi = 3.14$)

(3) A metallic sphere of diameter 20 cm. is melted and then recast in small spherical bails, each of radius 0.25 cm. Find the number of bails made.

(C) Solve any two (4)

(1) Find the value $\tan^2 30^0 + \cot^2 30 \sin^2 30 - \sec^2 45$

(2) Find the value $4\cot^2 45^0 - \sec^2 60^0 + \sin^2 30$

(3) Prove that $\dfrac{\sin\theta - \cos\theta + 1}{\text{Sin}\theta + \cos\theta - 1} = \dfrac{1 + \sin\theta}{\cos\theta}$

(D) Attempt any one (2)

(1) $y = a+b$ where a is constant and $b \propto x$, when $x = 27$ then $y = 61$ and when $x = 125$ then $y = 257$. Find y for $x = 9$

(2) If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$ show that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

(E) Answer the following (4)

(1) For equ. $6x^2 - x - 2 = 0$ .$\alpha$ and $\beta$ are two roots. Find $1/\alpha + 1/\beta$

(2) If the sum of the roots of equation $x^2 - x = (2x - 1)k$ is zero. Find the value of k

(3) Define : Transversal

(4) Define : Median of Triangle

Q.4

(A) In □ m ABCD, D – E – C and CE = 2DE, $\overrightarrow{AE}$ intersescts $\overline{BD}$ in P and the ray opposite to $\overrightarrow{CB}$ in Q. Prove that AQ = 4AP (4)

(B) Solve any two (6)

(1) A frequency distribution of life of 200 electric bulbs is given below. Find the median of grouped data.

| Life (in hours) of electrical bulbs | Number of bulbs |
|---|---|
| 400 – 499 | 32 |
| 500 – 599 | 31 |
| 600 – 699 | 42 |
| 700 – 799 | 36 |
| 800 – 899 | 33 |
| 900 – 999 | 26 |

(2) Frequency distrubution of marks obtained by 70 students in a test is given in following table. Complete the mean of the data.

| Marks | 0 | 1 | 2 | 3 | 4–6 | 7–9 | 10–14 | 15–19 | 20–24 | 25–32 |
|---|---|---|---|---|---|---|---|---|---|---|
| Frequ -ency | 5 | 3 | 3 | 2 | 5 | 15 | 20 | 10 | 6 | 1 |

(3) Simplify $\frac{x^8 - y^8}{x^6 - y^6}$

(C) Solve any two (4)

(1) In ΔABC m ∠B=90 AB=8 and BC=6 Find the radius of incircle.

(2) $\overline{AB}$ is a diameter of circle and $\overline{AC}$ is a chord of the circle other than a diameter. The tangent touching the circle at the point C intersect the ray opposite to BA in D. If m∠BAC = 40. Find m∠BDC

(3) In O (0,15) chords $\overline{AB}$ and $\overline{CD}$ cut each other at right angle at P. If AB=18 and CD=26, find OP.

(D) Solve any one (2)

(1) Find the roots of the equation $3x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

(2) Obtain the root of the equation $x^2 + 1 / x^2 = 7$

(E) Answer the following (4)

(1) Define : Interior of a circle

(2) Define : Tangent

(3) The numbers of representing surface area and the volume of a sphere are equal then r = ________

(4) 1 cu.mt = ___________ cu m.m.

Q.5

(A) $\overline{AB}$ isgiven. construct $\Delta$ PQR such that $\overline{QR} = \overline{AB}$ m $\angle P = 40$ and the length of median $\overline{PD} = 2/3$ AB. (steps not require) **(4)**

(B) Solve any two **(6)**

(1) The volume of a cylinder is in compound variation with the sequare of the radius and height. When the radius of the cylinder is 3.5 cm and height is 12 cm, the volume is 462 cm. Find the voulme of a cylinder whose height is 16 cm and the radius is 14 cm.

(2) By selling an item for Rs.56 gain is as much percent as it costs in rupees. What is the cost of the item?

(3) Solve the equation : $\left(\frac{2x+1}{x-1}\right)^2 - 5\left(\frac{2x+1}{x-1}\right) = 0$

(C) Solve any two **(4)**

(1) If the length of n side of an equilateral triangle is 10 untis. Find the area of triangle.

(2) In $\Delta$ ABC D $\in \overline{AB}$. CD = 6, BD = 9 and BC = 12 $\angle CAB \cong \angle BCD$. Find the perameter of $\Delta$ ADC

(3) In $\Delta$ PQR, QX is the bisector of Q and P – X – R. If PQ:QR = 3:5 and XR = 15 find PR.

(D) Solve any one **(2)**

(1) The radius and the height of a cone is 3.5 and 8.4 cm respectively. Find the area of the curved sufrece of the cone.

(2) $\overline{AD}$ is a median of $\Delta$ ABC. If AB=5 AC=7 and BC=10 then find AD

(E) For cyclic quadrilateral ABCD, AC is a bisector of $\angle A$ and CA is a bisector of $\angle C$. Prove that AC is a diameter of the circumcircle □ABCD **(4)**

**OR**

Answer the following

(1) □$^m$ PQRS is a cyclic quadrilateral. Find m$\angle$P.

(2) If $\odot$ (P,3) and $\odot$(Q,4) touch each other internally. Find PQ.

(3) " A circle has atleast one tangent at any point of the circle." State whether the sta ement is true or false.

(4) $\overset{\frown}{APB}$ and $\overset{\frown}{AQB}$ are respectively the minor arc and major arc of a circle then $\overset{\frown}{APB} \cap \overset{\frown}{AQB}$ = ________________

**SOLUTIONS**

**Suggestions**

(1) As shown in the marking scheme if the Calculation is correct up to final answer then the marks should be given according to the marks shown on the right side but if the whole sum is correct then full marks should be given.

(2) In the solution of the paper almost all the methods have been shown but if the sum is solved by other correct method then the marks should be given accordingly by marks distribution method.

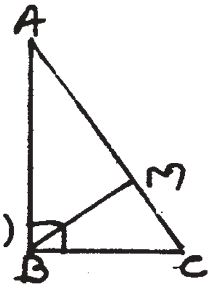
(3) If Given (Data), to Prove and figure are shown perfectly then one mark should be given.

(4) Necessary symbols like therefore(∴), Equal to (=), because (∵), congruent (≅), line-segment(-), ray(→) etc and necessary brackets are not shown properly then half mark should be cut from marks obtained from the sub-questions total marks.

(5) In theorem or any example if the names or letters are changed and answer is given than one mark should be cur from the total obtained all the sub-questions.

**1 (A)** If in a triangle the sum of the square of the length of any two side is equal to the square of the length of the third side, opposite side of that angle is right angle.

OR In $\Delta$ ABC, $AB^2 + BC^2 = AC^2$ than $\angle B$ is a right angle




**Given :** In$\Delta$ ABC B is a right angle and AC is a hypotenuse

**To Prove :** $AC^2 = AB^2 + BC^2$

**Proof :** In $\Delta$ ABC m B = 90 $\overline{AC}$ is a hypotenuse (given)

$\angle A$ and $\angle C$ are acute angles

Suppose M is the foot of the perpendicular drawn from B to on $\overline{AC}$

A – M – C ∴ AM + MC = AC - - - - - - (1)

Now $AB^2$ = AM x AC and $BC^2$ = MC x AC - (Th. 9)

$AB^2 + BC^2$ = AM x AC + MC x AC

= AC (AM + MC)

= AC x AC (From 1)

$AB^2 + BC^2 = AC^2$

**(B) (1)** $a^3 + b^3 + c^3 - 3abc$

Let $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$a^3 + b^3 + c^3 - 3abc$

$= (a+b)^3 - 3ab(a+b) + c^3 - 3abc$

$= (a+b)^3 + c^3 - 3ab(a+b) - 3abc$

$= (m)^3 + c^3 - 3ab(a+b+c)$

$= (m+c)\,(m^2-mc+c^2) - 3ab(a+b+c)$

$= (a+b+c)\,\{ (a+b)^2 - (a+b)c + c^2 \} - 3ab(a+b+c)$

$= (a+b+c)\,\{ a^2 + 2ab + b^2 - ac - bc + c^2 - 3ab \}$

$= (a+b+c)\,\{ a^2 + b^2 + c^2 - ab - bc - ca \}$

**(2)** $a^2(b+c) + b^2(c+a) + c^2(a+b) + 2abc$

$= a^2(b+c) + b^2c + ab^2 + ac^2 + bc^2 + 2abc$

$= a^2(b+c) + ab^2 + ac^2 + 2abc + b^2c + bc^2$

$= a^2(b+c) + a(b^2 + c^2 + 2abc) + bc(b+c)$

$= a^2(b+c) + a(b + c)^2 + bc(b+c)$

$= (b+c) + \{ a^2 + a(b + c) + bc \}$

$= (b+c) + \{ a^2 + ab + ac + bc \}$

$= (b+c) + \{ a(a+b) + c(a+b) \}$

$= (a+b)(b+c)(c+a)$

**(3)** $\frac{x+1}{X+2} \quad \frac{x-1}{x-2} \quad \frac{1}{X+1} \quad \frac{1}{x-1}$

$= \frac{(x-2)(x+1) + (x-1)(x-2)}{(x+2)(x-2)} + \frac{(x-1) + (x+1)}{(x+1)(x-1)}$

$= \frac{x^2 - x - 2 + x^2 + x - 2}{x^2 - 4} + \frac{2x}{x^2 - 1}$

$= \frac{(2x^2 - 4)}{x^2 - 4} + \frac{2x}{x^2 - 1}$

$= \frac{(2x^2 - 4)(x^2 - 1) + 2x(x^2 - 4)}{(x^2 - 4)(x^2 - 1)}$

$= \frac{2x^4 - 6x^2 + 4 + 2x^3 - 8x}{(x^2 - 4)(x^2 - 1)}$

$= \frac{2x^4 + 2x^3 - 6x^2 - 8x + 4}{(x+2)(x-2)(x+1)(x-1)}$

**(C) (1)** $(x^2 - 9)(y^2 - 25) - 60xy$

$= x^2y^2 - 25x^2 - 9y^2 + 225 - 60xy$

$= x^2y^2 - 30xy + 225 - 25n^2 - 30xy - 9y^2$

$= (xy - 15)^2 - (25x^2 - 30xy - 9y^2)$

$= (xy - 15)^2 - (5x + 3y)^2$

$= (xy - 15 + 5x + 3y)(xy - 15 - 5x - 3y)$

**(2)** $2x^3 + 3x^2 - 3x - 2$

$= 2x^3 - 2 + 3x^2 - 3x$

$= 2(x^3 - 1) + 3x(x-1)$

$= 2(x - 1)(x^2+x+1) + 3x(x-1)$

$= (x-1) \{ 2x^2 + 2x + 2 + 3x \}$

$= (x-1) \{ 2x^2 + 3x + 2 \}$

$= (x-1)(2x^2 + 3x + 2)$

$= (x-1)(2x^2 + 5x + 2)$

$= (x-1) \{2x^2 + 4x + x + 2 \}$

$= (x-1) \{2x(x+2) + 1(x + 2)\}$

$= (x-1)(x+2)(2x+1)$

**(3)** $81x^4 + 4$

$= 81x^4 + 36x^2 + 4 - 36x^2$

$= (9x^2 + 2)^2 - (6x)^2$
$= (9x^2 + 2 + 6x)(9x^2 + 2 - 6x)$
$= (9x^2 + 6x + 2)(9x^2 - 6x + 2)$

**(D)** **(1)** $\frac{x}{x+1} \quad \frac{x+1}{x+2} \quad \frac{x+2}{x+3} \quad \frac{x+3}{x+4} \quad \frac{x+4}{x+5} \quad \frac{x}{x+5}$

$= \frac{x}{x+1} \quad \frac{x+1}{x+2} \quad \frac{x+2}{x+3} \quad \frac{x+3}{x+4} \quad \frac{x+4}{x+5} \quad \frac{x+5}{x}$
$= 1$

**(2)** $\frac{a^2 + b^2 + c^2}{a^2 + ac + c^2} = \frac{b^2}{ac}$

$\therefore \frac{a^2 + b^2 + c^2}{a^2 + ac + c^2} = \frac{b^2}{ac} = \frac{a^2 + b^2 + c^2 - b^2}{a^2 + ac + c^2 - ac}$

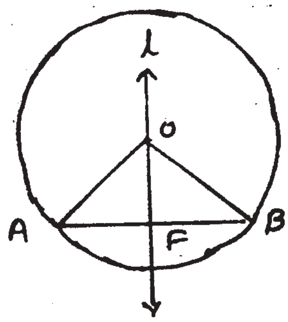
$\frac{a^2 + c^2}{a^2 + c^2}$

$= 1$

$\therefore b^2 / ac = 1$
$\therefore b^2 = ac$
b is a geometric mean of a and c

**(E)** **(1)** Ray **(2)** (-3,3) **(3)** ▭ **(4)** Flow chart

**2(A)** **Given :** $\overline{AB}$ is a chord of a Circle with centre O.
l is the perpendicular line From O to $\overline{AB}$




**To Prove :** l bisects $\overline{AB}$

**Proof :** OA = OB = r (radius of the circle)
O is a point equidistant from A and B.
Thus if $O \in \overline{AB}$, then O is the midpoint of $\overline{AB}$. So the perpendicular from O to $\overline{AB}$ bisects $\overline{AB}$. If $O \in \overline{AB}$ and F is the midpoint of $\overline{AB}$, then the perpendicular line from F to $\overline{AB}$ is the unique perpendicular bisector of $\overline{AB}$ and O being on the perpendicular bisector, OF is perpendicular line drawn from a point outside a line O is the line is unique.
$\overline{OF}$ =(l) bisects $\overleftrightarrow{AB}$.

**(B)** **(1)** In $\Delta ABC$ m $\angle A = 90$
$\overline{AD} \perp \overline{BC}$ and $D \in \overline{BC}$
$\Delta$ ABC is a right angle triangle
& $\Delta$ ADC is a also right angle triangle
In $\Delta$ ABC, COSC = CA/BC
IN $\Delta$ ADC, COSC = CD/CA

$\therefore \frac{CA}{BC} = \frac{CD}{CA}$

$\therefore CA^2 = BC \times CD$
$\therefore$ CA is a geometric mean of BC and CD

(2) $\frac{a^2 - a + 1}{b^2 - b + 1} = \frac{a^2 + a + 1}{b^2 + b + 1}$

$\therefore \frac{a^2 - a + 1}{a^2 - a + 1} = \frac{b^2 + a + 1}{b^2 + b + 1}$ (Alternendo)

$\therefore \frac{a^2 - a + 1 + a^2 + a + 1}{a^2 - a + 1 + a^2 - a + 1} = \frac{b^2 + b + 1 + b^2 + b + 1}{b^2 - b + 1 - b^2 - b - 1}$ (componendo & dividendo )

$\therefore \frac{2(a^2 + 1)}{-2a} = \frac{2(b^2 + 1)}{-2b}$

$\therefore \frac{a^2 + 1}{a} = \frac{b^2 + 1}{b}$

$\therefore a^2b + b = ab^2 + a$ (cross multiplication)

$\therefore a^2b - ab^2 + b - a = 0$

$\therefore ab(a-b) - 1(a-b) = 0$

$\therefore a - b = 0, \ ab - 1 = 0$

$\therefore a = b$ OR $ab = 1$

But $a \neq b \therefore a = 1/b$

$\therefore$ a is an inverse of b.

(3) Suppose height of cylinder x
volume of cylinder y
and radius of cylinder z

$X \propto y/z^2$

$x = Ky/z^2 \quad (K = 0)$

$\therefore 10 = K \times \frac{125.6}{4}$

$\therefore K = \frac{10 \times 4 \times 10}{1256}$

$\therefore K = 50/157$

Now $x = Ky/z^2$

$\therefore 8 = \frac{50 \times 9}{157 \times z^2}$

$\therefore z^2 = \frac{50 \times 9}{8 \times 157}$

$\therefore z^2 = \frac{225}{628}$

$\therefore z = \sqrt{225/628}$

$\therefore z = \frac{15}{2\sqrt{157}}$

Radius of cylinder is $15/2\sqrt{157}$ cm

**(C) (1)** x/5, x/4, x/3, x/2, x

M = value of the $\left(\frac{n+1}{2}\right)^{th}$ observation

$\therefore$ 8 = $(6/2)^{th}$ observation

$\therefore$ 8 = $3^{rd}$ observation

$\therefore$ 8 = x/3

$\therefore$ X = 24

$\therefore$ X = 24

$\therefore$ x/5 = 24/5 = 4.8

$\therefore$ x/4 = 24/4 = 6

$\therefore$ x/3 = 24/3 = 8

$\therefore$ x/2 = 24/2 = 12

$\bar{x} = \sum xi/n$

$= \frac{24 + 4.8 + 6 + 8 + 12}{5}$

$= \frac{54.8}{5}$

$\therefore \bar{x} = 10.96$

**(2)** $Z = 3M - 2\bar{x}$ $\quad$ x = 28.2

= 3(30.5) − 2(28.2) $\quad$ M = 30.5

= 91.5 − 56.4

$\therefore$ Z = 35.1

**(3)** 2/3 , 5/3, 1/3, 5/6, 1/6

$\bar{X} = \sum xi / n$

$= \frac{2/3 + 5/3 + 1/3 + 5/6 + 1/6}{5}$

= 11/3 5

$\therefore \bar{X} = 11/15$

**(D) (1)** f : N → Z $\quad f(x) = (-1)^x\ x$

$f(1) = (-1)^1 (1) = (-1)$

$f(2) = (-1)^2 (2) = 2$

$f(3) = (-1)^3 (3) = (-3)$

$f(4) = (-1)^4 (4) = 4$

$R_f$ = { ---- -3,-1,2,4 ----- }

**(2)** f(x) = (measures of the complementary angle of x)

f(0) = (measures of the complementary angle of 1) = 90

f(1) = (measures of the complementary angle of 1) = 89

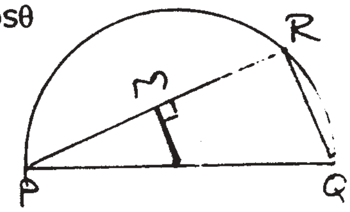
⋮ $\qquad$ ⋮

f(90) = (measures of the complementary angle of 90) = 0

$R_f$ = (0,90)

6

(E) (1) 1 (2) b/a = d/c (3) $x^{2/3}$ (4) 2cosθ




3 (A) **Given :** $\widehat{PQ}$ is a semi circle of a circle with centre
∠PRQ is an angle
Inscribed in the semi circle

**To Prove :** m∠PRQ = 90

**Proof :** $\widehat{PQ}$ is a semicircle

$\overline{PQ}$ is its corresponding chord and diameter

O is the midpoint of $\overline{PQ}$. If M is midpoint of $\overline{PR}$, $\overline{OM}$ is the line segment joining the centre and midpoint of PR

∴ $\overline{OM} \perp \overline{PR}$

∴ m ∠PMO = 90

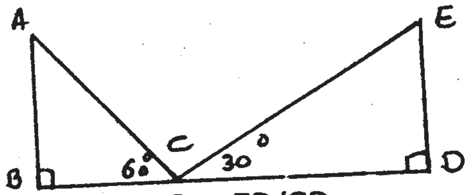
M and O are midpoints of $\overline{PR}$ and $\overline{PQ}$ respectively in Δ PRQ

$\overline{OM}$ || $\overline{RQ}$ and their transversal is $\overleftrightarrow{PA}$

∠PMO ≅ ∠PRQ (corroseponding angles)

m ∠PMO = m ∠PRQ = 90

∴ ∠PRQ is a right angle




(B) (1) AB = DE = Pillar = y mt.

∠ACB = $60^0$, ∠ECD = $30^0$

Distance between two pillar is 200 mt.

Let BC = x mt

CD = (200-x) mt

In Δ ABC, tanC = AB/BC

∴ tan30 = y/x

∴ 1/ $\sqrt{3}$ = y/x

∴ x = $\sqrt{3}$y --- (1)

In Δ ECD, tanC = ED/CD

∴ tan60 = y/200-x

∴ $\sqrt{3}$ = y/200-x

∴ $\sqrt{3} = \frac{y}{200 - 3y}$ (from 1)

∴ $\sqrt{3}$ 200 – 3y = y

∴ 4y = $\sqrt{3}$ 200

∴ y = $\sqrt{3}$ 50

∴ y = $\sqrt{3}$ x 1.73

∴ y = 86.50 mt

The height of pillars are 86.5 mt

(2) Area of minor sector = $\pi r^2\theta$ / 360

$= \frac{314 \times 10 \times 10 \times 90}{100 \times 360}$

= 78.5 sq cm

Are of minor sector = $\pi r^2\theta$ - Area of ΔAOB
360

= 78.5 – ½ bh

= 78.5 – ½ x 10 x 10

= 78.5 – 50

= 28.5sqcm.

7

**(3)** No of sphere = $\frac{\text{volume of sphere}}{\text{Volume of a bail}}$, $d_1=20cm$ $r_1=10$ cm

$r_2=0.25cm=1/4$ cm

$= \frac{4/3\pi \; r_1^3}{4/3\; \pi \; r_2^3}$

$= \frac{r_1^3}{r_2^3}$

$= \frac{10 \times 10 \times 10}{¼ \times ¼ \times ¼}$

= 64000 bails

**(C)** **(1)** $\tan^2 30^0 + \cot^2 30^0 \sin^2 60^0 - \sec^2 45^0$

$= (1/\sqrt{3})^2 + (\sqrt{3})^2 (\sqrt{3}/2)^2 - (\sqrt{2})^2$

$= 1/3 + 3 \times 3/4 - 2$

$= 1/3 + 9/4 - 2/1$

$= 7/12$

**(2)** $4\cot^2 45^0 - \sec^2 60^0 + \sin^2 30^0$.

$= 4(1)^2 - (2)^2 + (1/2)^2$

$= 4 - 4 + ¼$

$= ¼$

**(3)** $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1 + \sin\theta}{\cos\theta}$

LHS = $\frac{(\sin\theta - \cos\theta + 1)\ (\sin\theta + \cos\theta + 1)}{(\sin\theta + \cos\theta - 1)\ (\sin\theta + \cos\theta + 1)}$

$= \frac{(\sin\theta + 1 - \cos\theta)\ (\sin\theta + 1 + \cos\theta)}{(\sin\theta + \cos\theta - 1)\ (\sin\theta + \cos\theta\theta + 1)}$

$= \frac{(\sin\theta + 1)^2 - \cos^2\theta}{(\sin\theta + \cos\theta)^2 - 1}$

$= \frac{\sin^2\theta + 2\sin\theta + 1 - \cos^2\theta}{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}$

$= \frac{2\sin\theta + 2\sin^2\theta}{2\sin\theta\cos\theta}$

$= \frac{2\sin\theta\ (1 + \sin\theta)}{2\sin\theta\cos\theta}$

$= \frac{1 + \sin\theta}{\cos\theta}$

= RHS

$\therefore \frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1 + \sin\theta}{\cos\theta}$

**(D)** **(1)** $Y = a+b$, $b \propto x$ $b=Kx$

$\therefore Y = a + Kx$

$\therefore 61 = a + K(27)$

$\therefore 61 = a + 27K$ --- (1)

$y = a + Kx$

$257 = a + 125K$ --- (2)

$\therefore 61 = a + 27K$

$\therefore 257 = a + 125K$

$\therefore (-196) = (-98)K$

$\therefore K = 196/98$

$\therefore K = 2$

$\therefore 61 = a + 27(2)$ [K=2]

$\therefore 61 = a+54$

$\therefore a = 7$

Now $y = a + Kx$

$= 7 + 2(9)$

$= 7+ 18$

$y = 25$

**(2)** $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$(\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$

$\therefore \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$

$\therefore \cos^2\theta - 2\sin\theta\cos\theta = \sin^2\theta$

$\therefore \cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta = \sin^2\theta + \sin^2\theta$

$\therefore (\cos\theta - \sin\theta)^2 = 2\sin^2\theta$

$\therefore \cos\theta - \sin\theta = \sqrt{2}\sin\theta$

**(E)** **(1)** (-½) **(2)** ½

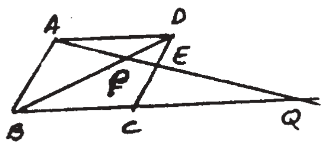
**(3)** **Transversal :** If $l_1$ and $l_2$ two distinct coplaner lines and if a line t intersescts $l_1$ and $l_2$ in two distinct point A and B, then t is called a transversal of $l_1$ and $l_2$

**(4)** **Median of a triangle :** A line segment, whose one end point is the vertex of triangle and the other end point is the midpoint of the opposite site, is called the median of the triangle.

**4 (A)**

**Given :** $\square^m$ ABCD, D – E – C and CE = 2DE, $\overleftrightarrow{AE}$ intersescts $\overline{BD}$ in P and the ray opposite to $\overrightarrow{CB}$ in Q.

**Prove :** AQ = 4AP




$\square^m$ ABCD, D – E – C and CE = 2DE (Given)

**DE/CE = 1/2** - - - - (1)

$\overrightarrow{CB}$ and $\overrightarrow{CQ}$ are opposite ray (Given)

$\overleftrightarrow{BC} = \overleftrightarrow{CQ}$

For $\Delta$ EDA and $\Delta$ ECQ

$\angle AED = \angle QEC$ (vertically opposite angles)

$\angle ADE = \angle QCE$ (alternative angle) of $\overline{AD} \parallel \overline{CQ}$ and $\overleftrightarrow{CD}$ their Transversal

$\Delta EDA \quad \Delta ECQ$ (A A)

$\therefore \frac{DE}{CE} = \frac{DA}{CQ}$

$\therefore \frac{DA}{CQ} = \frac{1}{2}$ (from 1)

$\therefore CQ = 2AD$

$CQ = 2BC$ (DA = BC, opposite side of $\square^m$ ABCD - - - - (2)

Now B-C-Q

$\therefore BQ = BC + CQ$

$\therefore BQ = BC + 2BC$ (from 3)

$\therefore BQ = 3BC$

$\therefore BQ = 3AD$ (AD = BC)

$\therefore BQ/AD = 3$ - - - - - - - - - - (3)

For $\Delta APD$ and $\Delta QPB$ $\angle APD \cong \angle QPB$ (vertically opposite angle)

$\angle APD \cong \angle QBP$ (alternative angle)

$\Delta APD \sim \Delta QPB$ (A A)

$\therefore AP/QP = AD/QB$

$\therefore \frac{QP}{AP} = \frac{QB}{AD}$

$\frac{QP}{AP} = 3$

$QP = 3AP$ - - - - - - - - - - (4)

But A – P – Q

$\therefore AQ = AP + PQ$

$\therefore AQ = AP + 3AP$ (From 4)

$\therefore AQ = 4AP$

**(B) (1)**

| **Life (in hrs) of Electric bulbs** | **No of bulbs** | **Cu. Freq** |
|---|---|---|
| 400 – 499 | 32 | 32 |
| 500 – 599 | 31 | 63 |
| 600 – 699 | 42 | 105 |
| 700 – 799 | 36 | 141 |
| 800 – 899 | 33 | 174 |
| 900 – 999 | 26 | 200 |

n = 200

M = value of (n/2)th observation

M = $100^{th}$ observation

L = 599.5, n/2 = 100, F = 63, f = 42, C = 100

$M = L + \frac{(n/2 - F)}{F} \times C$

$= 599.5 + \frac{(100 - 63) \times 100}{42}$

= 599.5 + 3700 / 42

= 599.5 + 88.1

$\therefore$ M = 687.6 hrs

10

(2)

| Marks | Fre(fi) | Midvalue xi | Fixi |
|---|---|---|---|
| 0 | 5 | 0 | 0 |
| 1 | 3 | 1 | 3 |
| 2 | 3 | 2 | 6 |
| 3 | 2 | 3 | 6 |
| 4 - 6 | 5 | 5 | 25 |
| 7 - 9 | 15 | 8 | 120 |
| 10 - 14 | 20 | 12 | 240 |
| 15 - 19 | 10 | 17 | 170 |
| 20 - 24 | 6 | 22 | 132 |
| 25 - 35 | 1 | 30 | 30 |
| n = 70 | | | ∑fixi = 732 |

$\bar{x} = \sum fixi / n$

$= 732/70$

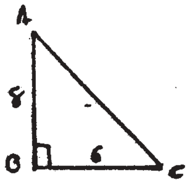
$\bar{x} = 10.46$ marks

(3) $\frac{x^8 - y^8}{x^6 - y^6}$

$= \frac{(x^4 + y^4)(x^2 + y^2)(x+y)(x-y)}{(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)}$

$= \frac{(x^4 + y^4)\ (x^2 + y^2)}{(x^2-xy+y^2)(x^2+xy+y^2)}$

(C) (1) Radius of incircle = $\frac{\text{Atitude + base - hypotenuse}}{2}$




$AC^2 = AB^2 + BC^2$

$\therefore AC^2 = 64 + 36$

$\therefore AC^2 = 100$

$\therefore AC = 10$

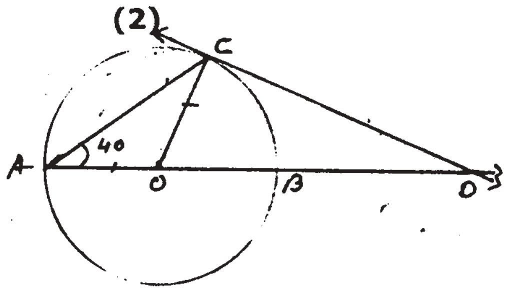
Radius of incircle = $\frac{\text{Atitude + base - hypotenuse}}{2}$

$= \frac{8 + 6 - 10}{2}$

$= 4/2$

$= 2$

(2)




$m\angle BAC = 40$

In $\Delta$ AOC OA = OC

$\therefore m\angle A = m\angle C = 40$

$\therefore m\angle 0 = 100$

$m\angle AOC + m\angle COB = 180$

$\therefore 100 + m\angle COB = 180$

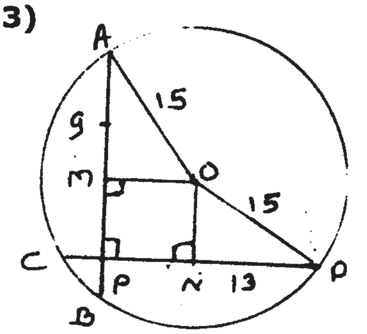
$\therefore m\angle COB = 80$

$m\angle OCD = 90$

In $\Delta$OCB, $m\angle ODB = m\angle BDC = 10$

11

(3)




$OM^2 = OA^2 - AM^2$

$= 15^2 - 9^2$

$= 225 - 81$

$\therefore OM^2 = 144$

$\therefore OM = 12$

$\therefore ON^2 = OD^2 - ND^2$

$= 225 - 169$

$= 56$

$\therefore ON = \sqrt{56}$

□OMPN is a rectangle ∴ $\overline{OP}$ is diagnal

But for ΔONP it's a hypotenuse

$OP^2 = PN^2 + ON^2$

$= 12^2 + ( 56)^2$

$= 144 + 56$

$\therefore OP^2 = 200$

$\therefore OP^2 = 100 \times 2$

$\therefore OP = 10\sqrt{2}$

(D) (1) $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

$\Delta = b^2 - 4ac$

$= (-2\sqrt{2})2 - 4(\sqrt{3})(-2\sqrt{3})$

$= 8+24$

$\Delta = 32, \ \Delta > 0$

$\alpha = \frac{-b + \sqrt{\Delta}}{2a}$

$= \frac{2\sqrt{2} + 4\sqrt{2}}{2\sqrt{3}}$

$= \frac{6\sqrt{2}}{2\sqrt{3}}$

$\therefore \alpha = \sqrt{6}$

$\beta = \frac{-b - \sqrt{\Delta}}{2a}$

$= \frac{2\sqrt{2} - 4\sqrt{2}}{2\sqrt{3}}$

$= \frac{-2\sqrt{2}}{2\sqrt{3}}$

$\therefore \beta = (-\sqrt{2}/3)$

The roots of the equations are $\sqrt{6}$ and $(-\sqrt{2}/3)$

(2) $x^2 + 1 / x^2 = 7$

$x^2 + 1 / x^2 + 2 = 7+2$ (adding 2 on both sides)

$\therefore (x + 1/x)^2 = 9$

$\therefore x + 1/x = \pm 3$

$\therefore X + 1/x = +3$ or $\therefore x + 1/x = -3$

$\therefore X^2 - 3x + 1 = 0$

$\therefore \Delta = b^2 - 4ac$

$= 9 - 4 \times 1 \times 1$

$= 9 - 4$

$\therefore \Delta = 5$

$\therefore X^2 + 3x + 1 = 0$

$\Delta = b^2 - 4ac$

$= 9 - 4 \times 1 \times 1$

$= 9 - 4$

$\therefore \Delta = 5$

12

$$\alpha = \frac{-b \pm \sqrt{\Delta}}{2a} \qquad \alpha = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$= \frac{3 \pm \sqrt{5}}{2} \qquad = \frac{-3 \pm \sqrt{5}}{2}$$

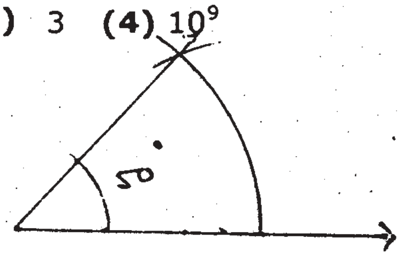
Roots of the equations are $\frac{-3 \pm \sqrt{5}}{2}$, $\frac{3 \pm \sqrt{5}}{2}$

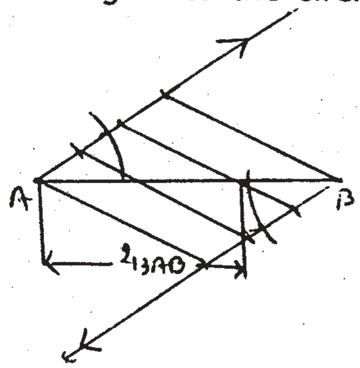
**(E)** **(1)** **Interior of circle** : The set of all points in the plane of circle lying at a distance less than the radiys from the centre of the circle is called the interior of the circle

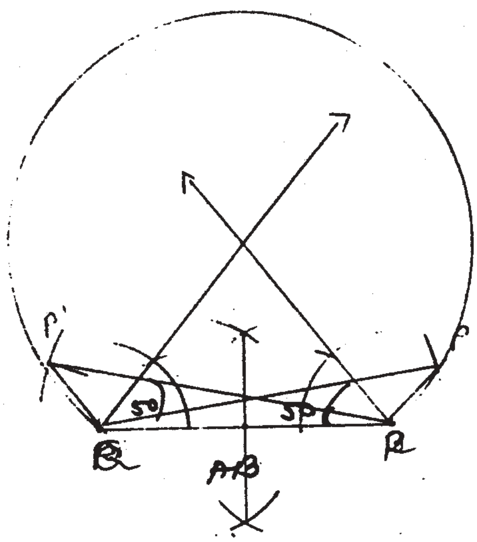
**(2)** **Tengent** : A line in the plane of a circle intersecting the circle at one and only point is called a tangent to the circle at that point

**(3)** 3 **(4)** $10^9$

5 (A)



| x | 462 | ? |
|---|---|---|
| Y | 7/2 | 14 |
| Z | 12 | 16 |

**5 (B)** **(1)** Suppose volume of cylinder x

radius of cylinder y

and height of cylinder z

$X \propto y^2z$

$\therefore X = Ky^2z$ (K = 0)

$\therefore 462 = K \times (7/2)^2 \times 12$

$\therefore K = \dfrac{462 \times 2 \times 2}{7 \times 7 \times 12}$

$\therefore K = 22/7$

Now $x = Kr^2h$

$= 22/7 \times 14 \times 14 \times 16$

$\therefore x = 9856$

$\therefore$ Volume of sphere is 9856 c.c.

**(2)** Let the cost of item is Rs x

Profit is x %

Profit at the cost of Rs x is $x^2/100$

Profit = S.P. – C.P.

$\therefore x^2/100 = 56 - x$

$\therefore x^2 + 100x - 5600 = 0$

$\therefore (x + 140)(x - 40) = 0$

$\therefore x = (-140)$ OR $x = 40$, $x>0$

$\therefore x = 40$

$\therefore$ Cost of item is Rs.40

**(3)** $12\left[\dfrac{2x+1}{x-1}\right]^2 - 5\left[\dfrac{2x+1}{x-1}\right] - 2 = 0$

Let 2x+1/x-1 = m

$\therefore 12m^2 - 5m - 2 = 0$

$\therefore 12m^2 - 8m + 3m - 2 = 0$

$\therefore 4m(3m-2) + 1(3m-2) = 0$

$\therefore 4m + 1 = 0$ OR $3m - 2 = 0$

$\therefore m = (-1/4)$ OR $m = 2/3$

now m = 2x+1 / x-1

$\therefore \dfrac{2x+1}{x-1} = \dfrac{-1}{4}$ OR $\therefore \dfrac{2x+1}{x-1} = \dfrac{2}{3}$

$\therefore 8x + 4 = -x + 1$ $\therefore 6x + 3 = 2x - 2$

$\therefore 9x = 3$ $\therefore 4x = -5$

$\therefore x = 1/3$ $\therefore x = -5/4$

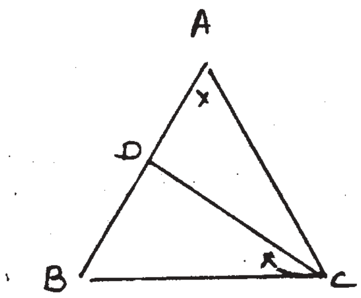
Thus roots of the equation are 1/3 or -5/4

**(C) (1)** Area of equilateral triangle = $\frac{\sqrt{3}a^2}{4}$ (a = 10)

$= \frac{\sqrt{3} \times 100}{4}$

$= \sqrt{3} \times 25$

$= 25\sqrt{3}$ sq. unit




**(2)** $\angle CAB \cong \angle BCD$ (given)

$\angle B \cong \angle B$

$\Delta ABC \sim \Delta CBD$

$AB/BC = \frac{BC}{BD} = \frac{AC}{CD}$

$\therefore \frac{AB}{12} = \frac{12}{9} = \frac{AC}{6}$

$\therefore AB = 144 / 9$

$\therefore AB = 16$

$\therefore AC = \frac{12 \times 6}{9}$

$\therefore AC = 72/9$

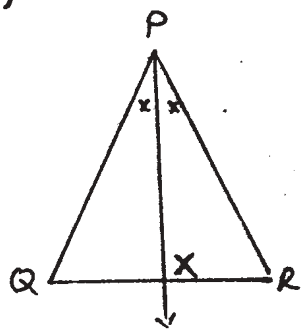
$\therefore AC = 8$

Perameter of $\Delta$ ABC = AD + CD + AC

= 7 + 6 + 8

= 21

**(3)**




$\therefore \frac{PQ}{PX} \quad \frac{QR}{XR}$

$\therefore \frac{PQ}{QR} \quad \frac{PX}{XR}$

$\therefore 3/5 = PX / 15$

$\therefore PX = 45/5$

$\therefore PX = 9$

PR = PX + XR

= 9 + 15

$\therefore$ PR = 24

**(D) (1)** r = 3.5 = 7/2 cm

h = 8.4 cm Curved surface area of cone = $\pi rl$

$l^2 = h^2 + r^2$

$= \frac{22 \times 7 \times 91}{7 \times 2 \times 10}$

$= (8.4)^2 + (3.5)^2$

$= \frac{1001}{10}$

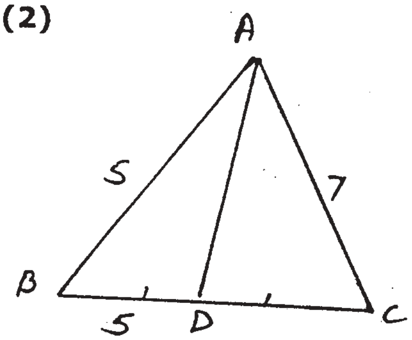
$= (0.7)^2 (12)^2 + (0.7)^2 (5)^2$

$= 100.1$ cm$^2$

15

$l^2 = (0.7)^2 (144 + 25)$
$l^2 = (0.7)^2 (169)$
$l = 0.7 \times 13$
$l = 9.1$ cm

(2)




$AB^2 + AC^2 = 2(AD^2 + BD^2)$
$5^2 + 7^2 = 2(AD^2 + 5^2)$
$25 + 49 = 2(AD^2 + 25)$
$74 = 2(AD^2 + 25)$
$37 = AD^2 + 25$
$AD^2 = 12$
$AD = 2\sqrt{3}$

(E) **Given :** for cyclic quandrilateral ABCD, $\overrightarrow{AC}$ is a bisector of $\angle A$ and $\overrightarrow{CA}$ is a bisector of $\angle C$

**To Prove :** AC is a diameter of circumcircle □ABCD

**Proof :** $\overrightarrow{AC}$ is a bisector of $\angle A$ (given)
$\therefore \frac{1}{2}\ m\angle DAB = m\angle BAC$ - - - (1)
$\overrightarrow{CA}$ is a bisector of DCB (given)
$\therefore \frac{1}{2}\ m\angle DCB = m\angle BCA$
$\therefore \frac{1}{2} m\angle DCB + \frac{1}{2} m\angle DAB = m\angle BAC + m\angle BCA$
But $m\angle BAC + m\angle BCA = 180$ ( **□ABCD is a cyclic quadrilateral)**
$\therefore \frac{1}{2} m\angle DAB + m\angle DCB = 180$
$\therefore \frac{1}{2} [m\angle DAB + m\angle DCB] = 180$
$\therefore m\angle DAB + m\angle DCB = 90$
In $\Delta ABC$ $m\angle A + m\angle B + m\angle C = 180$
$\therefore M\angle ABC + m\angle BAC + m\angle BCA = 180$
$\therefore m\angle ABC + 90 = 180$
$\therefore m\angle ABC = 90$
In $\Delta ABC$, $m\angle ABC = 90$
A, B, and C are on the same circle and a unique circle can pass through three points.
$\therefore$ The circle passing through A, B and C is only the circle passing through A, B, C and D
$\overline{AC}$ **is a diameter of the circumcircle of □ABCD**

**(E)** **(1)** 90 **(2)** 1 **(3)** True **(4)** {A, B}

**SOLUTIONS**

**Suggestions**

(1) As shown in the marking scheme if the Calculation is correct up to final answer then the marks should be given according to the marks shown on the right side but if the whole sum is correct then full marks should be given.

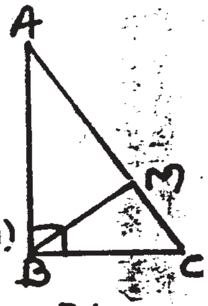
(2) In the solution of the paper almost all the methods have been shown but if the sum is solved by other correct method then the marks should be given accordingly by marks distribution method.

(3) If Given (Data), to Prove and figure are shown perfectly then one mark should be given.

(4) Necessary symbols like therefore(∴), Equal to (=), because (∵), congruent (≅), line-segment(-), ray(→) etc and necessary brackets are not shown properly then half mark should be cut from marks obtained from the sub-questions total marks.

(5) In theorem or any example if the names or letters are changed and answer is given than one mark should be cut from the total obtained all the sub-questions.

**1 (A)** If in a triangle the sum of the square of the length of any two side is equal to the square of the length of the third side, opposite side of that angle is right angle.




OR In Δ ABC, $AB^2 + BC^2 = AC^2$ than ∠B is a right angle

**Given :** InΔ ABC B is a right angle and AC is a hypotenuse

**To Prove :** $AC^2 = AB^2 + BC^2$

**Proof :** In Δ ABC m B = 90 $\overline{AC}$ is a hypotenuse (given)

∠A and ∠C are acute angles

Suppose M is the foot of the perpendicular drawn from B to on $\overline{AC}$

A – M – C ∴ AM + MC = AC - - - - - - (1)

Now $AB^2$ = AM x AC and $BC^2$ = MC x AC - (Th. 9)

$AB^2 + BC^2 = AM \times AC + MC \times AC$

$= AC (AM + MC)$

$= AC \times AC$ (From 1)

$AB^2 + BC^2 = AC^2$

**(B) (1)** $a^3 + b^3 + c^3 - 3abc$

Let $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$a^3 + b^3 + c^3 - 3abc$

$= (a+b)^3 - 3ab(a+b) + c^3 - 3abc$

$= (a+b)^3 + c^3 - 3ab(a+b) - 3abc$

$= (m)^3 + c^3 - 3ab(a+b+c)$

$= (m+c)(m^2-mc+c^2) - 3ab(a+b+c)$

$= (a+b+c)\{(a+b)^2 - (a+b)c + c^2\} - 3ab(a+b+c)$

$= (a+b+c)\{a^2 + 2ab + b^2 - ac - bc + c^2 - 3ab\}$

$= (a+b+c)\{a^2 + b^2 + c^2 - ab - bc - ca\}$