Std. : 10
Sub. : Mathematics

Q. 1 (A) State and prove geometric mean theorem. **(4)**

(B) Solve any two : **(6)**

(1) Simplify : $\frac{1}{x-1}+\frac{1}{x+1}+\frac{x-1}{x-2}+\frac{x+1}{x+2}$

(2) Factorize : $a^2b^2(a-b)+b^2c^2(b-c)+c^2a^2(c-a)$

(3) Factorize : $ab(a+b)+bc(b+c)+ca(c+a)+a^3+b^3+c^3$

(C) Factorize any two : **(4)**

(1) $(x+1)(x+2)(x+3)(x+4)-8$

(2) $1+a^4+a^5$

(3) $2x^4-x^3-3x^2+x+1$

(D) Solve any one : **(2)**

(1) Express in reduced form $\frac{x}{2x-4}-\left(\frac{x^2}{2}+\frac{4}{x}\right)\left(\frac{1}{x^3-2x+4}\right)$

(2) If $\frac{x^3-y^3}{x^3+y^3}=\frac{13}{4}$ find $\frac{x^2+y^2}{x^2-y^2}$.

(E) Fill in the blanks. **(4)**

(1) $f(x)=x^{-1}+4(-1)^x$, $f(4)=$ ..... $(-3, 2, \frac{17}{4})$

(2) P is a set of positive prime number of $f : P \to N$.
If $f(x)$ = {The number of factors of x} then Rf = ... [{1, 3}, {2}, {1, 2}]

(3) Loop is used in a flow-chart to indicate ..... steps in short.
(pictorial, repeated, algorithms)

(4) In a flow chart ..... symbol is used for input. (▭, ◇, ▱)

Q. 2 (A) Prove that the perpendicular drawn through the centre of a circle on a chord biscets the chord. **(4)**

(B) Solve any two : **(6)**

(1) If $\frac{a+b}{b+c}=\frac{c+d}{a+d}$ prove that either a = c or a + b + c + d = 0.

(2) If $\frac{a^2+a+1}{b^2+b+1}=\frac{a^2-a+1}{b^2-a+1}$ then prove that either a and b are equal or both of them are multiplicative inverse.

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(3) The expenditure of a tour is partly constant and partly varies directly as the number of students. When the number of students are 40 and 25, the expenditure per students are Rs. 75 and Rs. 90 respectively. If number of students are 50 then find the expenditure per student.

(C) Solve any two : (4)

(1) The mean of 90 data is 50. A data of class 30–40 is written 8 instead of 10. Find the correct mean.

(2) The sum and difference of the median and mean are 20 and 2 respectively. Find mode.

(3) For the data $x_1 < x_2 < x_3$ of mean and median are 30 and 20 respectively and $x_3 - x_1 = 50$ find data.

(D) Solve any one : (2)

(1) $f : A \to N$, $f(x) = \log_3 x$, if $R_f = \{1, 2, 3, 4\}$ find D_f.

(2) If $f(x) = ax^2 + b$ where $f(0) = -5$ and $f(2) = 7$ find $a + b$.

(E) Fill in the blanks. (4)

(1) $\frac{(2a-2)^3}{8(1-a)^3} + \frac{(2x-2)^4}{8(1-x)^4} + \frac{(2-2x)^5}{16(x-1)^5} =$ ..... $(-1, 0, 1)$

(2) Geometric mean of $a^2 - \frac{1}{b^2}$ and $b^2 - \frac{1}{a^2}$ is ..... $(1 - \frac{1}{ab}, ab - \frac{1}{ab}, ab - 1)$

(3) If $x + \frac{1}{y} \propto x - \frac{1}{y}$ then $x \propto$ ..... $(y, \frac{1}{y}, y^2)$

(4) $\sin 38 \cos 52 + \cos 38 \sin 52 =$ ..... $(0, 1, 2)$

Q. 3 (A) Prove that a tangent of a circle is perpendicular to the radius drawn through the point of contact. (4)

(B) Solve any two : (6)

(1) From a point at a height 3.75 m above the surface of a lake, the angles of elevation and depression of the top of a temple and its reflected image in the lake are 30° and 60° respectively. Find the height of the temple above the lake.

(2) The length of the diagonal of a square park is 42 m. Taking the point of intersection of the diagonals as teh centre and radius 21 m, two circular segments are added to the any two opposite sides. Find the area of the region of the square park together with the two added segments.

(3) A sector of a circle of radius 12 cm has the angle 120°. It is rolled up so that two bounding radii are joined together to form a cone. Find the volume of cone.

(C) Solve any two : (4)

(1) Prove that $\frac{2 + 2\sin\theta - 3\cos^2\theta}{\cos^2\theta} = \frac{3\sin\theta - 1}{1 - \sin\theta}$

(2) Evaluate : $2\cos^3 60 - 12\sin^4 60 + \frac{3}{4}\tan^2 30 + 12\cot 45$

(3) If $\tan\theta + \sec\theta = 1.5$ then find $\sin\theta$.

(D) Solve any one : (2)

(1) If $x^4 + 9y^2 \propto 3x^2y$ then establish the relation between x and y under variability.

(2) Prove that $(\cos\theta + \sec\theta)^2 + (\sin\theta + \text{cosec}\,\theta)^2 = 5 + (\tan\theta + \cot\theta)^2$

(E) Answer the following. (4)

(1) Define Altitude of a triangle.

(2) Define Adjacent line segment.

(3) The roots of $ax^2 + bx + c = 0$ are $\frac{3}{4}$ and $\frac{4}{3}$ find $\frac{a+b+c}{abc}$.

(4) The difference of roots of $ax^2 + bx + c = 0$ is 4 find Δ (Delta).

Q. 4 (A) In ΔABC, $\overline{AM} \perp \overleftrightarrow{BC}$. If $\angle C$ is an obtuse angle then prove that $AB^2 = AC^2 + BC^2 + 2BC.MC$. (4)

(B) Solve any two : (6)

(1) If the mean of the following grouped data is 16.6 then find k.

x	8	12	15	k	20	25	30
f	12	16	20	24	16	8	4

(2) Find mode of the following grouped data :

Class	60–64	65–69	70–74	75–79	80–84	85–89
Frequency	13	28	35	12	9	3

(3) Simplify : $\left[2 + \frac{x}{2}\left(1 - \frac{x}{x-2}\right)\right] + \left[\frac{1}{x+2}\left(\frac{x^2}{2-x} - 1\right)\right]$

(C) Solve any two : (4)

(1) If the distance between the two chords of measure 24 and 18 lying different half plane of a diameter is 21 then find diameter of the circle.

(2) $\overline{AB}$ is a diameter of a circle with centre O. $\overline{OD}$ is a radius perpendicular to $\overline{AB}$. If C is a distinct point of the same circle then find the possible measures of $\angle ACD$.

(3) The adjacent sides of a cyclic parallelogram are 15 and 8. Find the radius of the circle.

(D) Solve any one : (2)

(1) If one positive root of $x^2 + bx + 8 = 0$ is twice of other then find b.

(2) Solve : $\sqrt{3}x^2 - 2\sqrt{2}x = 2\sqrt{3}$

(E) Answer the following : (4)

(1) Define : Concyclic points.

(2) Define : Angle in a segment of a circle.

(3) What is the distance of a chord 19.2 from the centre of a circle of diameter 20.

(4) Total surface area of a hemisphere is 462 find its diameter.

Q. 5 (A) Construct ΔXYZ such that hypotenure $YZ = ST$ and $XY = \frac{1}{3} ST$. Where $ST = 10$ cm. Write the all steps of constructions. (4)

(B) Solve any two : (6)

(1) If $x^2 - xy + y^2 \propto x^2 + xy + y^2$ prove $x \propto y$.

(2) Find the roots of $\left(x + \frac{1}{x}\right)^2 = 4 + \frac{3}{2}\left(x - \frac{1}{x}\right)$

(3) The number obtained by adding the squares of a rational number and its reciprocal is greater by $\frac{7}{4}$ than the sum of the rational number and its reciprocal. Find the number.

(C) Solve any two : (4)

(1) In Δ ABC, A–M–B, A–N–C. $\overleftrightarrow{MN} \parallel \overleftrightarrow{BC}$. If AM = 2x, MB = 24, AN = 3 and NC = 18 find AB.

(2) In Δ ABC, $m\angle B = 90$, $\overline{BM} \perp \overline{AC}$, A–M–C. If AB = 20 and BC = 15 find BM.

(3) $\square^m$ ABCD, if $AB^2 + BC^2 = 170$, AC = 12 find BD.

(D) Solve any one : (2)

(1) The curved surface area of a cone is 550 cm^2 and its slant height is 25 cm, find its volume.

(2) The volume of a sphere is $\frac{6336}{7}$ cm^3 find its curved surface area.

(E) For a cyclic quadrilateral ABCD, $\overrightarrow{AC}$ is a bisector of $\angle A$ and $\overrightarrow{CA}$ is a bisector of $\angle C$. Prove that $\overline{AC}$ is a diameter of the circumcircle of $\square$ABCD. (4)

OR

(E) (1) What is the difference of circumcircle and in circle formed in a right angle triangle where sides forming right angle are 5 and 12.

(2) For $\overset{\frown}{PQ}$ of $\odot$ (O, 4), $m\angle POQ = 45$. Find the major $\overset{\frown}{PQ}$.

(3) $\overline{AB}$ is a diameter and $\overline{AC}$ is a chord of the circle. The tangent at C touches $\overrightarrow{AB}$ in D. If $\angle BDC = 40$ find $\angle BAC$.

(4) Two congruent circles of centres P and Q have chords AB and CD at equal distance respectively. If $m\angle PAB = 40$ find $m\angle CQD$.

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