Std. : X
Sub. : Mathematics

Q. 1 (A) If for a correspondance between two triangles, two pairs of corresponding sides are proportional and the inclued angles are congruent, then the correspondance is a similarity. Prove it. **(4)**

(B) Solve any two : **(6)**

(1) Simplify : $\frac{(x^2-y^2)^3+(y^2-z^2)^3+(z^2-x^2)^3}{(x-y)^3+(y-z)^3+(z-x)^3}$

(2) Factorize : $x^4(y^2-z^2)+y^4(z^2-x^2)+z^4(x^2-y^2)$

(3) Factorize : $2ab(a+2b)+2bc(b+2c)+2ca(c+2a)+9abc$

(C) Factorize any two : **(4)**

(1) $4x^2-y^2+2x-2y-3xy$ (2) $2x^3+5x^2-4x-3$

(3) $(x^2-4)(y^2-9)-24xy$

(D) Solve any one : **(2)**

(1) Express in reduced form $\frac{1}{8-8x}-\frac{1}{8+8x}+\frac{x}{4+4x^2}-\frac{x}{2+2x^4}$

(2) If $\frac{x^3+27x}{9x^2+27}=\frac{172}{171}$, find x.

(E) Fill in the blanks : **(4)**

(1) If $f: R^- \to R$, $f(x)=\frac{|x|}{x}$, then Rf = ... ({1}, {−1}, {−1, 1})

(2) If $f: R^+ \to R$, $f(x)=\frac{1}{2}\log_y x$, then $f(3)$ = ... $\left(1, \frac{1}{2}, \frac{1}{4}\right)$

(3) Loop is used in a flow-chart to indicate ..... steps in short.
(pictorial, repeated, alphanumerical)

(4) Control unit is a part of ..... (C.P.U., I.P.D., O.P.D.)

Q. 2 (A) Prove that in the congruent circles, conguent chords are equidistant from the centres of the circles. **(4)**

(B) Solve any two : **(6)**

(1) If $\frac{a^2-bc}{x}=\frac{b^2-ca}{y}=\frac{c^2-ab}{z}$, then prove that a = b = c.

OR $(a+b+c)(x+y+z)=ax+by+cz$.

(2) If $\frac{x^2+y^2-p^2}{x^2yz} = \frac{z^2+2xy}{y^2zx} = \frac{2z^2-p^2}{z^2xy}$, then prove that each ratio is equal to $\left(\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx}\right)$.

(3) The monthly salary of an employee partially varies directly as the number of years of service put in by him and is partially constant. After 5 years of service, his salary is Rs. 4325 and after 8 years of service, the salary is Rs. 4700. Find the number of years of services put by him, when his salary is Rs. 5700.

(C) Solve any two : (4)

(1) The mean of 20 observation was calculated to be 24 instead of 23.5. On reachecking it was found that the wrong observation which was copied is seven–fifth the correct observation. Find the correct observation.

(2) For a data $\overline{X} - M = 2$ and $\overline{X} + M = 20$, find the mode.

(3) The observations of a data are $\frac{x}{3}$, $\frac{x}{5}$, $\frac{x}{6}$ and $\frac{x}{4}$, where $x > 0$. If the median of the dta is 18, then find it's mean.

(D) Solve any one : (2)

(1) If $f : Z \to Z$, $f(x) = x^2 - x - 2$, then find the value of $\frac{1}{x}[f(x+1) - f(x)]$

(2) If $f : N \to Z$, $f(x) = (-1)^x.x + x$, then find the range of f.

(E) Fill in the blanks : (4)

(1) $\frac{(x-1)^3}{(1-x)^3} \times \frac{(x-2)^4}{(2-x)^4} + \frac{(x-3)^5}{(3-x)^5} =$ ..... {1, 0, 1}

(2) If ab = 6, bc = 12, ca = 8, then a + b + c = ... {9, 12, 15}

(3) If $\sin 2\theta = \cos 3\theta$, then θ = ... {0°, 90°, 18°}

Q. 3 (A) Prove that an angle inscribed in a semicircle is a right angle. (4)

(B) Solve any two : (6)

(1) Two vertical pillars are situated at the two opposite vertices of a square field. The height of one is double the other. The angle of elevation of the top of one pillar from the bottom of the other are complementary. If the height of the larger pillar is 40 m. Find the sides of the square field.

(2) The angle subtended at the centre of a circle of radius 21 c.m. by the minor segment of the circle is 120°. Find the area of the major segment.

(3) An inverted conical vessel whose internal radius is 5 c.m. and height 8 c.m. is full of water. When lead shots each of radius 0.5 c.m. are dropped into the vessel, one–fourth of the water flows out. Find the number of spherical lead shots dropped into the vessel.

(C) Solve any two : (4)

(1) Prove $(2\cos^2\theta - 1)^3 + 3(1 - 2\sin^2\theta) = 4(\cos^6\theta - \sin^6\theta)$

(2) Evaluate : $2\cot^2 45° + \cos^3 60° - 2\sin^4 60° + \frac{3}{4}\tan^2 30°$

(3) If $\tan\theta - \cot\theta = \operatorname{cosec}\theta$, $0° < \theta < 90°$, find θ.

(D) Solve any one : (2)

(1) If $x^2y^2 + 4 \propto xy$, then prove that $x \propto \frac{1}{y}$.

(2) Prove that $\sqrt{\sec^2 A + \operatorname{cosec}^2 A} = \tan A + \cot A$.

(E) Answer the following : (4)

(1) Define : Adjacent line-segment

(2) Define : centroid

(3) Find 'c' if the equation $x^2 - 5x + 6 = 0$ and $x^2 + 3x + c = 0$ have a common root.

(4) If $x + \frac{1}{x} = 2$, find $x^7 - \frac{1}{x^7}$.

Q. 4 (A) In right angled Δ ABC, AC is the hypotenuse. D and F are respectively the mid-points of $\overline{BC}$ and $\overline{AB}$. Prove that $4(AB^2 + CF^2) = 5AC^2$. (4)

(B) Solve any two : (6)

(1) Express as rational expression : $\left[\frac{x+3}{x+2} \cdot \frac{x^2-1}{x+6}\right] - \left[\frac{x^2+7}{2} - \frac{x^2+3}{x} \cdot \frac{4x}{3}\right]$

(2) Find the median of :

Class	400–799	800–1199	1200–1599	1600–1999	2000–2399	2400–2799
f	6	15	28	7	2	2

(3) The information collected on throwing a balanced dice 50 times is tabulated below :

Number	1	2	3	4	5	6
Frequency	x	7	9	12	10	y

If $x + y = 12$ and the mean is 3.42, find the value of x and y.

(C) Solve any two : (4)

(1) In Δ ABC, $\angle B$ is a right angle. AB = 5 and BC = 12. Find the radius of the incircle of ΔABC.

(2) $\overset{\frown}{PQ}$ and $\overset{\frown}{XY}$ are arcs of a circle with centre O. If PQ = XY and $m\angle PQO = 25°$. Find $m\angle XOY$.

(3) In $\odot(O, 15)$ chords $\overline{AB}$ and $\overline{CD}$ intersects each other at right angles at P. If AB = 18 and CD = 26, find OP.

(D) Attempt any one : (2)

(1) Solve for x : $2\left(x + \frac{1}{x}\right)^2 - 3\left(x + \frac{1}{x}\right) - 8 = 0$.

(2) For what value of k the equation : $x^2 - 2(5 + 2k)x + 3(7 + 10k) = 0$ has reciprocal roots.

(E) Answer the following : (4)

(1) Define : Segment fo a circle

(2) Define : Exterior of a circle.

(3) In a cyclic quadrilateral ABCD, $m\angle A = 2m\angle C$ and $m\angle B = 3m\angle D$ then $m\angle A$ = ... and $m\angle D$ = ...

(4) A cone and a hemisphere have equal base radii and equal volume. The ratio of their heights is ..... .

Q. 5 (A) $\overline{XY}$ is given. Construct ΔABC such that AB = 2XY, $m\angle C = 30^\circ$ and the length of the median AD is $\frac{3}{5}$XY. Write the steps of construction. (4)

(B) Solve any two : (6)

(1) If $x - \frac{1}{y} \propto x + \frac{1}{y}$, prove that $x \propto \frac{1}{y}$ $(x \neq 0, y \neq 0)$

(2) If α, β are the roots of the equation $x^2 + x + 1 = 0$, find the equation whose roots are $\frac{\alpha+1}{\beta}$ and $\frac{\alpha}{\beta+1}$.

(3) One-fourth of a herd of camels was seen in forest. Twice a square-root of the herd had gone to mountain and remaining 15 camels were on the bank of river. Find the total number of camels.

(C) Solve any two : (4)

(1) In Δ ABC, the bisector of $\angle A$ intersects BC at D. If AB = 10. BC = 18. AC = 12.5 then find DC.

(2) $\Delta ABC \sim \Delta PQR$, $\overline{AM} \perp \overline{BC}$, $\overline{PN} \perp \overline{QR}$, $M \in \overline{BC}$, $N \in \overline{QR}$. If the ratio of the area of Δ ABM and Δ PQN is 9 : 25 and AM = 6 then find PN.

(3) In Δ ABC, $m\angle B = m\angle A + m\angle C$. If AB = 8 and BC = 15, find the median BD.

(D) Solve any one : (2)

(1) The minute hand of a clock is 7 c.m. Find the area swept by the minute hand in 5 min. ($\pi = 3.14$)

(2) The volume of a sphere is double it's surface area. Find it's diameter.

(E) $\overline{AB}$ is a diameter of a circle and $\overline{AC}$ is a chord other than the diameter of the circle. The tangents at A and C intersect each other at the point D in the exterior of the circle. Prove that $m\angle ADC = 2\, m\angle BAC$. (4)

OR

(E) (1) $\overline{XY}$ is a chord of $\odot$ (P, 10). If XY = 19.2 find the distance of the chord from the centre.

(2) $\overline{AB}$ is the diameter of $\odot$ (P, 20). If $\angle ACB$ is an angle in the semicircle and $BC = 20\sqrt{3}$, then find $m\angle A$.

(3) If $\odot$ (A, 5) and $\odot$ (B, 8) touch each other, find the possible measures of AB.

(4) $\overline{AB}$ and $\overline{CD}$ are the chords of a circle with centre O. If AB = 3.5, CD = 3.5, $m\angle ABO = 30^\circ$, then find $m\angle COD$.

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