Std. : 10
Sub. : Mathematics

Q. 1 (A) A line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points and there fore it cuts two line segments lying in the same half plane of the line are proportional to the two sides of the triangle.
State and prove it converse. (4)

(B) Solve any two : (6)

(1) Simplify : $\dfrac{19x+4}{4+\dfrac{3}{4+\sqrt{x}}} - \dfrac{13x-4}{4-\dfrac{3}{4-\sqrt{x}}}$.

(2) Factorise : $(x-a)^2(b-c)+(x-b)^2(c-a)+(x-c)^2(a-b)$

(3) Factorise : $(a+b)(a^2-ab+b^2)+(b+c)(b^2-bc+c^2)+(c+a)(c^2-ac+a^2)-6abc$

(C) Factorize : (Any Two) (4)

(1) $(a^2-6a-36)^2+179(a^2-6a-36)+66a^2$

(2) $(x+9)(x+2)(x+3)(x+4)-8$

(3) $x^4+4x^3+6x^2+4x+1$

(D) Solve : (Any One) (2)

(1) Express in reduced form : $\dfrac{1}{8-8x} - \dfrac{1}{8+8x} + \dfrac{x}{4+4x^2} - \dfrac{x}{2+2x^4}$

(2) If $\dfrac{x^3+27x}{9x^2+27} = \dfrac{172}{171}$ find the value of x.

(E) Fill in the blanks : (4)

(1) If $R \to Z$, $f(x) = |x|$ = greatest integer of x not greater than x then $f(2.3)$ = ... (1, 2, 3)

(2) $f : N \to R$, $f(x) = x^{-1} + 4(-1)^x$ then $f(4)$ = ... $\left(4\frac{1}{4},\ -3\frac{1}{4},\ 2\frac{1}{4}\right)$

(3) Loop is used in a flow-chart to indicate ..... steps in short.
(pictorial, repeated, alphanumerical)

(4) Control unit is a part of ..... . (C.P.U., I.P.D., O.P.D.)

Q. 2 (A) In the congruent circles, congruent chords are equidistant from the centres of the circles. State its converse and prove it. (4)

(B) Solve : (Any Two) (6)

(1) If $(a+b+c+d)(a-b-c+d) = (a+b-c-d)\,a-b+c-d)$
Prove that a, b, c, d are in proportion.

(2) If a, b, c, d are in proportion then prove that
$abcd\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}+\frac{1}{d^2}\right) = a^2+b^2+c^2+d^2$

(3) Profit of running a travel bus varies as number of passenger than a fixed number. When the number of passengers are 50 and 60 the profits are Rs. 1,000 and Rs. 1,250. What would be the profit when there are 40 passengers.

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(C) Solve : (Any Two) **(4)**

(1) If the observations of ungrouped data are a, b, c where $a < c < b$ and the mean and mode are 15 and 18 respectively and also $a - b + 11 = 0$ then find a, b, c.

(2) Mean of 10 observations is 104.8 and $\Sigma u_i f_i = 4.8$ and C = 10, find assumed mean.

(3) x, $2x$, $4x$ are the observations of the data. If the mean of the data is greater than median by 2 find mode.

(D) Solve : (Any One) **(2)**

(1) If $Z \to Z$, $f(x) = x^2 - x - 2$ find the value of $\frac{1}{x}[f(x+1) - f(x)]$

(2) If $R \to R$, $f(x) = x^2 - x$ then find the value of $f(x+1) - f(-x)$.

(E) Fill in the blank. **(4)**

(1) $\frac{(x-1)^3}{(1-x)^3} \times \frac{(x-2)^4}{(2-x)^4} + \frac{(x-3)^5}{(3-x)^5} =$ ..... (–1, 0, 1)

(2) If $\frac{a}{2} = \frac{b}{3} = \frac{c}{4}$ then $3a + 2b - 3c =$ ..... (0, 1, 3)

(3) $\sin\theta \cdot \sec(90 - \theta) =$ .....: (1, $\tan\theta$, $\sin\theta \cdot \cos\theta$)

(4) $1 + \cot^2(90 - \theta) =$ ..... ($\tan^2\theta$, $\sec^2\theta$, $\operatorname{cosec}^2\theta$)

Q. 3 (A) Prove that A tangent to a circle is perpendicular to the radius drawn from the point of contact. **(4)**

(B) Solve : (Any Two) **(6)**

(1) The two temples are situated on the opposite bank of river. One temple is 40 m. high, observing from the top of this temple the angle of depression of the top and bottom of the opposite temple are 12° 30' and 21° 48'. Find the breadth of the river and height of second temple.
tan 12° 30' = 0.2217, tan 21° 48' = 0.4000

(2) A semi-circular dome is placed over a cone shaped minaret. Radius of the base of the minaret and the radius of the dome is 7 m. Slant height of the minaret is 30 m. If the cost of painting from outside is Rs. 15 per sq. m., find the total cost of painting.

(3) Water flows from a cylindrical pipe of radius 0.2 c.m. at the rate of 10 metre/minute. How much time would be required to fill a conical tank of a diameter 40 c.m. and height 24 c.m. ?

(C) Solve : (Any Two) **(4)**

(1) If $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$ then prove that $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

(2) Prove that $\tan^2 A \cdot \sec^2(90 - A) - \sin^2 A \cdot \operatorname{cosec}^2(90 - A) = 1$

(3) Evaluate : $\frac{1}{3}\sin^2 60 - \frac{1}{2}\sec 60 \cdot \tan^2 30 + \frac{4}{3}\sin^2 45 \cdot \tan^2 60$

(D) Solve : (Any One) **(2)**

(1) If $x \propto \cos^4\theta - \sin^4\theta$, $y \propto \cos^2\theta - \sin^2\theta$ then prove that $x \propto y$.

(2) Prove that : $\frac{\sin^2 60}{1 - \cos 60} + \frac{\sin^2 60}{1 - \cos 60} = 2$

(E) Answer the following : (4)

(1) Define : Adjacent line segment.

(2) Define : Centroid.

(3) The roots of the equation $x^2 + (m - 1)x - 4 = 0$ are opposite to each other then $m =$ ..... .

(4) Find the product ofthe root $2x^2 + 5\sqrt{3}\,x + 6 = 0$.

Q. 4 (A) In right angled ΔABC, AC is the hypotenuse. D and F are respectively the mid-points of $\overline{BC}$ and $\overline{AB}$. Prove that $4(AB^2 + CF^2) = 5AC^2$ (4)

(B) Solve : (Any Two) (6)

(1) Simplify : $\dfrac{\frac{x^3}{8} - \frac{8}{x^3}}{\left(\frac{x}{2} - \frac{2}{x}\right)\left(\frac{x}{2} + \frac{2}{x} - 1\right)} \times \dfrac{\frac{1}{2} - \frac{1}{x}}{\frac{1}{x^2} + \frac{1}{4} + \frac{1}{2x}}$

(2) If the mean of the following frequency distribution is 122.7 find the missing frequency.

Class	60–79	80–99	100–119	120–139	140–159	160–179	180–199
Frequency	7	4	x	18	5	5	2

(3) Find the mean of following observations :

Value	–2 to 2	3 to 7	8 to 12	13 to 17	18 to 22	23 to 27
Frequency	3227	4096	2048	512	64	3

(C) Solve : (Any Two) (4)

(1) In ΔABC $\angle B$ is a right angle. AB = 5 and BC = 12. Find the radius of the in circle of ΔABC.

(2) In $\odot$ (P, 8) chords $\overline{AB}$ and $\overline{CD}$ intersect each other at right angle at M. If AB = 14, CD = 12 then find PM.

(3) In $\odot$ (P, r) $\overline{AB}$ is a chord $\overline{PM} \perp \overline{AB}$. $\overrightarrow{PM}$ intersects the circle in point N. If AB = 30, MN = 9 then find the diameter of the circle.

(D) Solve : (Any One) (2)

(1) Find the solution in R : $8\left(x^2 + \frac{1}{x^2}\right) - 42\left(x - \frac{1}{x}\right) + 29 = 6$

(2) Find the solution in R : $x^2 - 59x = 1992$.

(E) Answer the following : (4)

(1) Define : Segment of a circle.

(2) Define : Tangent of a circle.

(3) Δ ABC inscrifed in $\odot$ (P, r). If $m\angle APB = 124$ and $m\angle BPC = 154$ then find the measures of all the three angles.

(4) Find area of a segment of a circle formed by a chord equal to the diameter.

Q. 5 (A) In $\odot (P, \sqrt{5})$ construct a regular hexagon inscribed in it.
Write the steps of contruction. (4)

(B) Solve : (Any Two) **(6)**

(1) If $\frac{a}{b} \propto x + y$, $\frac{b}{c} \propto x - y$ and $\frac{c}{a} \propto x^2 + y^2$ then prove that $x^4 - y^4$ is constant.

(2) Find the roots of the equation : $2\left(\frac{x-1}{2x+1}\right)^2 - 5\left(\frac{x-1}{2x+1}\right) = 12$

(3) There are in all 40 boys and girls in a class. Anilbhai distributes chocolates in such a way that each girl gets number of chocolates equal to the number of boys and each boy gets no. of chocolates equal to the number of girls. If the total 768 chocoltes are distributed, then find the numbr of boys and girls.

(C) Solve : (Any Two) **(4)**

(1) In Δ ABC the bisector of ∠A intersects BC at D. If AB = 10 and BC = 18, AC = 12.5 then find DC.

(2) In ΔABC m∠B = m∠A + m∠C. If AB = 8, BC = 15 then find median BD.

(3) A motor-car goes 10 km. in the north from there it goes 12 km. in the east and then 6 km. in the north. How far is the motor-car now from its starting point.

(D) Solve : (Any One) **(2)**

(1) The minute hand of a clock is 7 a.m. Find the area swept by minute hand in 5 min. ($\pi = 3.14$)

(2) Water is supplied to a population of 2,000 persons at the rate of 308 litre per head. A conical tank supplying water has a radius 14 m. and height 15 m. if tank is filled once with water for how many days would the quantity of wate last ?

(E) $\overline{PQ}$ and $\overline{PR}$ the two contruent chords of a circle. If a line *l* touches the circle at the point P then prove that *l* || QR. **(4)**

OR

(E) (1) $\overline{XY}$ is a chord of ⊙ (P, 10). If XY = 19.2 find the distance of chord from the centre. **(4)**

(2) $\overline{AB}$ is a diametr of ⊙ (P, 20). If ∠ACB is an angle in the semicircle and BC = $20\sqrt{3}$ then find m∠A.

(3) If ⊙ (A, 5) and ⊙ (B, 8) touch each other, find the possible measures of AB.

(4) $\overline{AB}$ and $\overline{CD}$ are the chords of a circle with centre O. If AB = 3.5, CD = 3.5, m∠ABO = 30° find m∠COD.

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