STD X

M.M 100

Maths (SET- A)

Time:3 Hrs.

Q.1.(A) Prove that if the correspondence between two triangles, two pairs of corresponding sides are proportional and the included angles are congruent, then the correspondence is a similarity. [4]

Q.1.(B) Solve : [Any 2] [6]

1. $a^3 + b^3 + c^3 - 3abc$
2. $(a-b)(a+b)^2 + (b-c)(b+c)^2 + (c-a)(c+a)^2$
3. Simplify :

$$\frac{x^2-3x+2}{x^3-8} + \frac{x^2-9}{x^2+7x+12} \times \frac{x^3+2x^2+4x}{x^2+3x-4}$$

Q.1.(C) Solve : [Any 2] [4]

1. Factorize : $3x^3 + 4x^2 - 5x - 2$
2. Factorize : $42(x+y)(x-y) - 13xy$.
3. Factorize : $6(a-c)^2 - 25(a-b)(b-c)$

Q.1.(D) Solve : [Any 1] [2]

1. Simplify : $\frac{2(x-1)(2-x)(x-3)}{(1-x)(2x-6)(x-2)}$
2. If $ab=1$, prove that $\frac{1+a}{1+b} = \frac{a-1}{1-b}$

Q.1.(E) Fill in the blanks choosing the correct alternative : [4]

1. For $f: N-\{1\} \to N$, $f(x)$ = greatest prime factor of x, $f(35) - f(20) =$ _____
   (1, 2, 0)
2. The graph of $f:[1,4] \to [0,3]$, $f(x) = 4-x$ is _____
   (line, ray, line-segment)
3. _____ unit of CPU of a computer instructs to perform different procedures in proper order.
   (Memory, control, Arithmetical – logical)
4. Symbol ▭ is used in a flow – chart for _____.
   (start, procedure, input)

Q.2.(A) Prove that in congruent circles the chords equidistant from the centers are congruent. [4]

Q.2.(B) Solve : [Any 2] [6]

1. If $\frac{a-b}{c} = \frac{b-c}{a} = \frac{c-a}{b}$ $(a,b,c, \neq 0)$, then prove that a=b=c or a+b+c=0.
2. If $a^3 + b^3 : a^3 - b^3 = 91 : 37$. Find the value of a : b
3. The volume of a cone varies directly as its height when its radius is constant and varies directly as the square of its radius when height is constant. The volume of a cone with radius 3cm and height 7cm is 66 cu.cm. Find the volume of the cone with radius 7cm and height 3cm.

Q.2.(C) Solve [Any 2] [4]

1. Observations of some data are $\frac{x}{5}$, $\frac{x}{4}$, $\frac{x}{2}$ and $\frac{x}{3}$ where x>0. If the median of the data is 8, find the value of x. What will be the mean of the data.
2. For a unimodal grouped data, M- $\bar{x}$ =2 and z=20.5. then find M.
3. Find the median of the following grouped data:

Score	1	2	3	4	5
frequency	15	22	27	10	6

Q.2.(D) Solve [Any 1] [2]

1. Find the range of f:{2,4,8,16} → N f(n) = $\log_2$ n.
2. For f: Z→Z, $f(x) = 2x^2-3x+4$. Find the value of f(2) – f(-1).

Q.2. (E) Fill in the blanks choosing correct alternative: [4]

1. If x = 1000, then $25x^2 - 5x + 1 - \frac{125x^3}{5x+1}$ = _____. $\left[\frac{1}{501}, \frac{1}{5001}, \frac{1}{5100}\right]$
2. If 2x =3y and 5y = 8z, then x:y:z = _____. [2:3:8, 3:5:8, 12:8:5]
3. If $4x^2y^2+1 = 4xy$, then x ∝ _____. [y^2, $\sqrt{y}$, 1/y]
3. Sin38° . cos52° + cos38° . sin52° = _____.(0, 1, 2).

Q.3. (A) Prove that a tangent of a circle is perpendicular to the radius drawn through the point of contact. [4]

Q.3. (B) Solve [Any 2] [6]

1. Watching from the top and the base of a 100metre high tower, the angles of elevation of the top of a hill are found to be 45°

and 56°19′ respectively. Find the height of the hill. (tan 56°19′ = 1.5000)

2. The diameter of a conical tent is 14 metres and its height is 24 metres. How much canvas will be required to make the tent, if the width of the canvas is 1.10 metres? What will be the cost of the canvas at the rate of Rs. 3.50 per metre?
3. The radius and thickness of a circular gold are 4cm and 0.5 cm respectively. The medal is melted and small spherical balls of radius 2 millimetre are made out of it. Find the number of balls thus obtained.

Q.3. (C) Solve [Any 2] [4]

1. Evaluate :

$$2\cot 45° + \cos^3 60° - 2\sin^4 60° + \frac{3}{4}\tan^2 30°$$

2. Prove that :

$$\frac{3\cos^2\theta - 2\sin\theta - 2}{\cos^2\theta} = \frac{1 - 3\sin\theta}{1 - \sin\theta}$$

3. Prove that :

$$\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1 + \sin\theta}{\cos\theta}$$

Q.3. (D) Solve [Any 1] [2]

1. The cube of x varies inversely as the square root of y. x=2, when y=9. Then find x, when $y = \frac{1}{81}$
2. If $\frac{\cot\theta + \tan\theta}{\sec\theta} = 2$, find the value of θ

Q.3. (E) Answer the following : [4]

1. If the roots of $ax^2+bx+c = 0$ differ by 3, then prove that discriminant $\Delta = 9a^2$.
2. If the roots of $16x^2 - (k-1)x-1=0$ are opposite numbers, find the value of K.
3. Define :similarity of triangles.
4. Define : Altitude of a triangle.

Q.4. (A) Medians $\overline{AD}$ and $\overline{BE}$ of ΔABC intersect each other at G. A line through G parallel to $\overline{BD}$ intersect $\overline{AC}$ at K. Prove that AC = 6EK.

Q.4. (B) Solve [Any 2] **[6]**

1. **Simplify :**

$$\frac{16a^2-(b-c)^2}{(4a+c)^2-b^2}+\frac{b^2-(c-4a)^2}{(4a+b)^2-c^2}+\frac{c^2-(4a-b)^2}{(b+c)^2-16a^2}$$

2. **Find the mean of the following frequency distribution :**

Class	1	2	3	4	15-25	26-46	47-69	70-100
frequency	2	4	6	8	16	25	15	4

3. **Find the mean, median and mode of the observations**

$\frac{3}{5}, \frac{5}{3}, \frac{4}{3}, \frac{5}{6}$, and $\frac{7}{6}$

Q.4. (C) Solve [Any 2] **[4]**

1. **In ΔABC, $\angle B$ is a right angle. If AB = 5 and BC – 12 then find the in radius of ΔABC.**
2. **In $\odot(P, 6.5)$, the parallel chords $\overline{AB}$ and $\overline{CD}$ are lying on the opposite sides of a diameter .If AB = 12 and the distance between the two chords is 8.5. Find CD.**
3. **$\overline{AB}$ is a diameter of a circle and $\overline{AC}$ is a chord other than a diameter. The tangent touching the circle at C intersects the ray opposite to $\overrightarrow{BA}$ in D. If $m\angle BAC = 40$. Find $m\angle BDC$.**

Q.4. (D) Solve [Any 1]: **[2]**

1. **Solve :** $\frac{1}{x-2}+\frac{1}{x+3}=\frac{7}{2x}$
2. **If one root of $x^2 - 9x + c = 0$ is twice the other, find C and the solution of the equation.**

Q.4.(E) Answer the following : **[4]**

1. **Define : Circle.**
2. **Define : angle subtended by the minor arc at the centre.**
3. **The ratio of radii of two spheres is 1:3 . Then the ratio of their surface areas is _____.**
4. **The volume of a sphere is $\frac{4}{3}\pi$ cu.cm. then its diameter is _____.**

Q.5. (A) $\overline{AB}$ is given. Construct ΔPQR such that QR = AB. $m\angle P = 75$ and the length of the altitude PM = ½ AB. Write the steps of construction. **[4]**

Q.5. (B) Solve [Any 2] [6]

1. A rail track of length l_1 is at a temperature T_1. On being heated to temperature T_2, the length increases to l_2. The extension $l_2 - l_1$ in the length is in compound variation as the initial length l_1 and the increase $T_2 - T_1$ in the temperature. The constant of variation for the increase in the length of the rail track is 12×10^{-6}. If the temperature rises from 10° C in winter to 40° C in summer, find the extension in the length of a 4000km long rail track.

2. Solve in R : $2\left[x^2 + \frac{1}{x^2}\right] - 9\left[x + \frac{1}{x}\right] + 14 = 0$

3. A family spends Rs. 200 per month for buying groundnut oil. With the increase of Rs. 10 per Kg in the price of oil, they could purchase 1 kg of oil less. Find the original price of oil per Kg.

Q.5. (C) Solve [Any 2] [4]

1. In trapezium ABCD, $\overline{AD} \parallel \overline{BC}$ and $\overline{AC} \cap \overline{BD} = \{P\}$. If PD = 2, PB = 6 and AP = 3. Find CP.

4. The lengths of sides of a triangle are 6, 8 and 10. The perimeter of another triangle similar to this triangle is 36. Find the lengths of the sides of the second triangle.

3. In trapezium PQRS, $\overline{PQ} \parallel \overline{RS}$ and PS=QR. The distance between PQ and RS is 12. PQ = 7 and RS = 17. Find QR.

Q.5. (D) Solve [Any 1]: [2]

1. In ΔABC, $m\angle B = 90$ and $BM \perp AC$. if BM = 4.8, AC = 10 and AB<BC. Find AM.

A cone is formed from a semicircle with radius 21cm. Find the area of the curved surface of the cone.

Q.5 (E) Lines l and m passing through a point T lying in the exterior of a circle with centre O touch the circle respectively at A and B. If $m\angle ATB = 90°$. Prove that □ OATB is a square.

OR

Q.5. (E) Answer the following : **[4]**

1. $\overset{\frown}{PQ}$ and $\overset{\frown}{XY}$ are the arcs of the circle with centre O. If PQ = XY and $m\angle PQO = 25$. Find $m\angle XOY$.
2. The bisector of $\angle A$ of ΔABC intersects the circumcircle at D. If $m\angle BCD = 37$. Find $m\angle BAC$
3. $\overline{PB}$ is a radius of $\odot(P,6)$ and $\overleftrightarrow{AB}$ is a tangent to the circle. If $m\angle PAB = 30$, then PA = _____.
4. In a cyclic quadrilateral ABCD, $m\angle A = 3m\angle C$. Find $m\angle A$.