Time: 3 hours   Q. Paper set I   MATHS – I (050)(E)   MARKS – 75

XII – Science   ①

Secondary School

Q-1(A) (1) Obtain the formula for the area of the ΔABC, where $A(x_1, y_1), B(x_2, y_2)$ & $C(x_3, y_3) \in R^2$ (3)

(2) Find the area of the triangle with the vertices (5, 3), (4, 5) & (3, 1) by shifting the origin at (5, 3). (1)

Q-1(B) Calculate any two (4)

(1) Find the points which divide the line segment joining (0, 0) and (a, b) in to n equal parts

(2) Prove that the coordinates of all three vertices of an equilateral triangle cannot be rational numbers

(3) If (1, −1), (−4, 4) and (3, 6) are the vertices of rhombus, find its coordinates of fourth vertex.

Q-1(C) Calculate any two (4)

(1) Area of ΔABC is 4. Coordinates of A and B are resp. A(2, 1) and B(4, 3). Find the coordinates of C if it lies on line $3x - y - 1 = 0$.

(2)

(2) A line passes through $(\sqrt{3}, -1)$ and the length of the segment perpendicular to it from the origin is $\sqrt{2}$. Find the eq^n of the line.

(3) Find the eq^n of the lines passing through $(4, 5)$ and parallel to and perpendicular to $2x + y - 1 = 0$

Q-1(D) Obtain p-α form of a line (3)

(3)

Q-2(A) (1) What do you mean by con-current lines? Obtain the necessary and sufficient condition for three lines in R^2 to lie concurrent?

(2) The cartesian eq^n of $\overleftrightarrow{AB}$ is $4x - 3y + 10 = 0$. If one parametric eq^n is $x = 3t - 1, t \in R$ then obtain the second parametric eq^n. (1)

Q-2(B) (1) Obtain the general form of the eq^n of a circle in R^2. Obtain the condition for this eq^n to represent a circle and find its centre and radius (2)

(2) Find the measure of angle between the lines $6x^2 - xy - y^2 = 0$ (1)

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(3) Find the combined eq^n of lines through the origin which are perpendicular to lines $ax^2+2hxy+by^2=0$

Q.2(C) (i) Show that points of intersection of the lines representation by $2x^2-5xy+2y^2+7x-5y+3=0$ with the axes lie on a circle, Find the eq^n of this circle (3)

OR

(i) Find the eq^n of the circle which is orthogonal to the circles $x^2+y^2-6x+1=0$ and $x^2+y^2-4y+1=0$ and whose centre lies on the line $3x+4y+6=0$ (2)

(ii) Find the set of intersection of the circle $x^2+y^2=25$ & the line $x+y-7=0$ (1)

Q.2(D) Prove that the eq^n of the lines through the origin which make an angle of measure of α and $x+y=0$ is $x^2+2xy\sec 2\alpha+y^2=0$ $\left(0<\alpha<\frac{\pi}{4}\right)$ (3)

OR

In ΔABC, A is $(4,-3)$ and two of the medians lie along the lines $2x+y+1=0$ and $x+5y-1=0$. Find the coordinates of B and C

(4)

Q-3(A) (1) Define the conic section and hence write the condition that conic section becomes a parabola. only write the eqn of focus and directrix (2)

(2) If a focal-chord of the parabola $y^2 = 4ax$ forms an angle of measure θ with the positive direction of the x-axis, then show that its length is $4a\operatorname{cosec}^2\theta$ (2)

OR

Find the set of points P so that

(1) the sum of the slopes of tangents drawn to the parabola from P is a constant k.

(2) The product of slopes of the tangents drawn to the parabola from P is a constant k.

Q-3(B) (1) If P is on the ellipse and S and S' are the focii then prove that $SP + S'P = 2a$ (2)

(2) If the line containing the chord joining α and β passes through the focus $(ae, 0)$ then prove that $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{e-1}{e+1}$

OR

(5)

points P and Q are corresponding points of ellipse and auxillary circle resp. if the line parallel to OQ and passing through the point P intersects the axes in E and F resp, then P.T $PE = b$ and $PF = a$

Q-3(C) (1) Explain the auxillary circle and the eccentric angle of the hyperbola (2)

(2) For a point on hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ prove that $SP \cdot S'P = CP^2 - a^2 + b^2$ (2)

Q-3(D) (1) Find the standard eq^n of hyperbola whose focus is $(3, 0)$ and $b = 2$ (1)

(2) Find the length of the chords of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ cut on the axes. (2)

Q-4(A) Which curve is represented by the eq^n $x^2 + 2xy + y^2 + \sqrt{2}x - \sqrt{2}y = 0$?

Obtain the eq^n of the curve in the standard form. Find the coordinates of the foci, the eq^n of directrix, the length of axes and the eccentricity?

OR

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(A) Determine the following curves by converting it in standard form:

(1) $x^2+y^2-4x-6y-2=0$

(2) $xy=16$

(B) (1) State the Schwartz inequality in R^3. From this, obtain the triangular inequality (2)

(2) Show that for any $a \in R$, the direction of $(2,3,5)$ and $(a, a+1, a+2)$ cannot lie same or opposite.

(C) (1) By vector method, obtain position vector of incentre of triangle (2)

(2) If G is the centroid of $\triangle ABC$ and P is any point in plane of this triangle, then P.T

$$\vec{PA}+\vec{PB}+\vec{PC} = 3\vec{PG}$$ (2)

(D) (1) A river flows with a speed of 5 km/s. One desire to cross the river in direction $\perp^{ar}$ to the flow. Find in what direction he swim if his speed is 8 km/s (2)

(1) Find a unit vector in R^2 which is $\perp^{r}$ to $(1,3)$

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Q-5(A) (1) prove that the necessary condition for two distinct lines $\bar{r} = \bar{a} + k\bar{l}$; $k \in R$ and $\bar{r} = \bar{b} + k\bar{m}$; $k \in R$ in R^3 to intersect each other is $(\bar{a} - \bar{b}) \cdot (\bar{l} \times \bar{m}) = 0$ (2)

(2) If the length of the ⊥r from the origin to the plane is p and the direction angles of ⊥r are α, β, γ then show that the eq^n of the plane is $x\cos\alpha + y\cos\beta + z\cos\gamma = p$

OR (2)

Obtain vector eq^n and cartesian eq^n of the plane passing through two parallel lines.

(B) (1) obtain the necessary and sufficient conditions for the eq^n $x^2+y^2+z^2+2ux+2vy+2wz+d = 0$ represent a sphere (1)

(2) Find the intersection of the lines

$$\frac{x-3}{1} = \frac{y+2}{-1} = \frac{z-1}{1} \quad \&$$

$$\frac{x}{2} = \frac{y+1}{0} = \frac{z+3}{3} \qquad (3)$$

OR

Find the eq^n of the line ⊥r to $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passing through $(3, -1, 11)$

(8)

(C) (1) Find the coordinates of a point equidistant from (a, 0, 0), (0, b, 0), (0, 0, c) and (0, 0, 0) (2)

(2) verify that the eqn

$$3x^2+3y^2+3z^2-3x-2y-6z-66=0$$

represents the sphere, or not. If it represents the sphere then find the centre and radius of sphere. (2)

(D) Obtain the eqn of the plane containing $\bar{r}=(1,1,1)+k(2,1,2)$, $k\in R$ and passing through (1, -1, 2)

OR

(D) Find the length, the foot and the eqn of ⊥r from (2, -1, 2) to the plane $2x-3y+4z-44=0$

Solution of paper set I

MATHS - I

PAPER ÷ 4 (1)

Q I (A) (1) Theo. Text page - 9
Ch - I

Ans (2) Shifting the origin at (5, 3),

$\therefore$ New co.ord of (5, 3) are

$(5-5, 3-3) = (0, 0)$

New co.ord of (4, 5) are

$(4-5, 5-3) = (-1, 2)$

New co.ordinates of (3, 1) are

$(3-5, 1-3) = (-2, -2)$

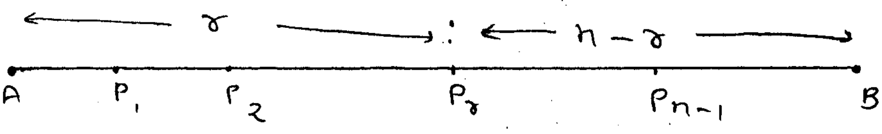
$$\therefore D = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = \begin{vmatrix} -1 & 2 \\ -2 & -2 \end{vmatrix} = 2+4 = 6$$

$\therefore$ The area of the triangle

$= \frac{1}{2}|D| = \frac{1}{2}|6| = 3$ units

Q I (B)

(1) Let $P_1, P_2, \cdots, P_r, \cdots P_{n-1}$ be the points which divide $\overline{AB}$ in n equal parts.



Let $P_r = P_r(x_r, y_r)$, then P_r divides $\overline{AB}$ in ratio $r : n-r$ from A (see figure) $r = 1, 2, \cdots, (n-1)$

$$x_r = \frac{\frac{r}{n-r} \cdot a + 0}{\frac{r}{n-r} + 1} \qquad y_r = \frac{\frac{r}{n-r} \cdot b + 0}{\frac{r}{n-r} + 1}$$ ②

$$x_r = \frac{ar}{r+n-r} \qquad y_r = \frac{br}{r+n-r}$$

$$\therefore x_r = \frac{ar}{n}, \qquad y_r = \frac{br}{n}$$

$\therefore$ The required points of division are
$P_r(x_r, y_r) = P_r\left(\frac{ra}{n}, \frac{rb}{n}\right) \quad r = 1, 2, \cdots (n-1)$.

(2) Let $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ be the vertices of an equilateral triangle and if possible let all $x_i, y_i \in Q$, for $i = 1, 2, 3$.

$\therefore AB = BC = AC = a$ and
$x_1, y_1, x_2, y_2, x_3, y_3 \in Q \quad \therefore a^2 \in Q$
$(\because a^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 \in Q)$

$\therefore$ Area of $\triangle ABC = \Delta = \frac{1}{2} |D|$

where $D = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Since all $x_i, y_i \in Q$, $D \in Q$
$\therefore \Delta \in Q$

But $\Delta = \frac{1}{2} bc \sin A = \frac{1}{2} \cdot a \cdot a \sin 60$

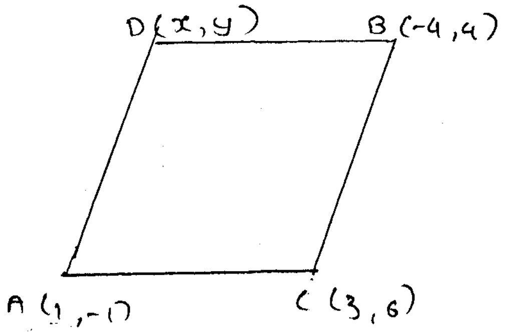
(In equilateral triangle $m\angle A = 60$)

$= \frac{\sqrt{3}}{4} a^2 \notin Q \quad (\because a^2 \in Q, \sqrt{3} \notin Q)$

Thus $\Delta \in Q$ and $\Delta \notin Q$ which are contradictory statements. ③

$\therefore$ All the co-ordinates of all the vertices of an equilateral ΔABC can not be rational.

(3)



Let $A(1,-1)$, $B(-4,4)$, $C(3,6)$ then

$AB^2 = (1+4)^2 + (-1-4)^2$
$= 25 + 25 = 50$

$BC^2 = (-4-3)^2 + (4-6)^2$
$= 49 + 4 = 53$

$AC^2 = (1-3)^2 + (-1-6)^2$
$= 4 + 49 = 53$

Since $AC = BC$, construct parallelogram $ABCD$. This will be rhombus due to $AC = BC$. Let $D(x, y)$.

Now mid-point of $\overline{CD}$ = mid point of $\overline{AB}$

$\therefore \left(\frac{x+3}{2}, \frac{y+6}{2}\right) = \left(\frac{1-4}{2}, \frac{-1+4}{2}\right)$

$\therefore x = -6$, $y = -3$

$\therefore D(x,y) = D(-6,-3)$ is the fourth vertex of the rhombus

Q1 (c) (4)

(1) The co-ordinates of any point C on line $3x - y - 1 = 0$ can be taken as $(x, 3x-1)$ $(\because y = 3x-1)$

$\therefore$ Area of $\Delta ABC = 4$

$\therefore \frac{1}{2}|D| = 4$ $\therefore D = \pm 8$

where $D = \begin{vmatrix} x & 3x-1 & 1 \\ 2 & 1 & 1 \\ 4 & 3 & 1 \end{vmatrix} = 4x$

$\therefore 4x = \pm 8$ $\therefore x = \pm 2$

For $x = 2$, point $C(x, 3x-1) = C(2, 5)$

For $x = -2$, point $C(x, 3x-1) = C(-2, -7)$

(2) Length of the perpendicular from origin on the line is given $\sqrt{2}$.

$\therefore$ The equation of line is $p-\alpha$ form is

$x \cos\alpha + y \sin\alpha = p = \sqrt{2}$

The line passes through $(\sqrt{3}, -1)$

$\therefore \sqrt{3}\cos\alpha - \sin\alpha = \sqrt{2}$

$\therefore \frac{\sqrt{3}}{2}\cos\alpha - \frac{1}{2}\sin\alpha = \frac{1}{\sqrt{2}}$

$\therefore \cos\alpha \cos\frac{\pi}{6} - \sin\alpha \sin\frac{\pi}{6} = \cos\frac{\pi}{4}$

$\therefore \cos\left(\alpha + \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{4}\right)$

$\therefore \alpha + \frac{\pi}{6} = \pm\frac{\pi}{4}$

$\therefore \alpha = \frac{\pi}{4} - \frac{\pi}{6}$ or $\alpha = -\frac{\pi}{4} - \frac{\pi}{6}$

$\therefore \alpha = \frac{\pi}{12}$ or $\alpha = -\frac{5\pi}{12}$ $(\because -\pi < \alpha \leq \pi)$ (5)

$\therefore \cos\alpha = \cos\frac{\pi}{12}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\therefore \sin\alpha = \sin\frac{\pi}{12}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\cos\alpha = \cos\left(-\frac{5\pi}{12}\right)$

$= \sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\sin\alpha = \sin\left(-\frac{5\pi}{12}\right)$

$= -\frac{\sqrt{3}+1}{2\sqrt{2}}$

$\therefore$ Required equations are

(1) $\frac{\sqrt{3}+1}{2\sqrt{2}}x + \frac{\sqrt{3}-1}{2\sqrt{2}}y = \sqrt{2}$

$\therefore (\sqrt{3}+1)x + (\sqrt{3}-1)y = 4$

(2) $\frac{(\sqrt{3}-1)}{2\sqrt{2}}x - \frac{(\sqrt{3}+1)}{2\sqrt{2}}y = \sqrt{2}$

$\therefore (\sqrt{3}-1)x - (\sqrt{3}+1)y = 4$

(3) The equations of lines parallel to and perpendicular to $2x+y=1$ are respectively $2x+y=k$ and $x-2y=k'$ both these lines are passing through $(4,5)$.

$\therefore 2(4)+5 = k$ | $\therefore 4-2(5) = k'$
$\therefore k = 13$ | $\therefore k' = -6$

$\therefore$ The required lines are respectively

$2x+y = 13$ and $x-2y+6=0$

Q.1.(D) Theory Text book page No. 41.

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Q2 (A) (1) Ch-3 theory Page : 49-50

Ans : 2

(2) Substituting $x = 3t-1$ in $4x - 3y + 10 = 0$, we get

$4(3t-1) - 3y + 10 = 0$

$\therefore 3y = 12t + 6$

$\therefore y = 4t + 2$, $t \in R$ is the second parametric equation

Q2 (B) (1) Theorem circle Ch: 4

(2) Here $a = 6$, $h = \frac{1}{2}$, $b = -1$

$$\therefore \theta = \tan^{-1} \frac{2\sqrt{\frac{1}{4} + 6}}{|6-1|}$$

$$= \tan^{-1} 1 = \frac{\pi}{4}$$

(3) $ax^2 + 2hxy + by^2 = 0$ is the combined equation of a pair of lines.

$$\therefore b\left(\frac{y}{x}\right)^2 + 2h\frac{y}{x} + a = 0$$

Put $\frac{y}{x} = m$, then m is the slope of the lines represented by equation (1)

Let m_1 and m_2 be the roots of $bm^2 + 2hm + a = 0$, then m_1 and m_2 are also the slopes of two seperate lines represented by (1)

$\therefore m_1 + m_2 = \frac{-2h}{b}$ and $m_1 \cdot m_2 = \frac{a}{b}$

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The lines whose slopes are $m_1' = \frac{-1}{m_1}$ and $m_2' = \frac{-1}{m_2}$ are perpendicular to the lines represented by (1)

Now, $m_1' + m_2' = -\frac{1}{m_1} - \frac{1}{m_2} = -\left(\frac{m_1 + m_2}{m_1 m_2}\right)$

$$= -\frac{-2h/b}{a/b} = \frac{2h}{a}$$

$\therefore m_1' \cdot m_2' = \left(-\frac{1}{m_1}\right)\left(-\frac{1}{m_2}\right) = \frac{1}{m_1 m_2} = \frac{b}{a}$

$\therefore$ The combined equation of lines with slopes m_1' and m_2' is

$$y^2 - (m_1' + m_2')\, xy + m_1' \cdot m_2' \, x^2 = 0$$

$$\therefore y^2 - \frac{2h}{a} xy + \frac{b}{a} x^2 = 0$$

$$\therefore ay^2 - 2hxy + bx^2 = 0$$

$$\therefore bx^2 - 2hxy + ay^2 = 0$$

$\therefore$ This is the required equation.

Q2 (c)

(i) Put $y = 0$ to find the intersection of pair of lines with x-axis,

$$2x^2 + 7x + 3 = 0$$

$$\therefore (x+3)(2x+1) = 0$$

$$\therefore x = -3 \; , \; x = \frac{-1}{2}$$

$\therefore$ The pair of lines intersect x-axis at $A(-3, 0)$, $B(-\frac{1}{2}, 0)$

Similarly putting $x = 0$, we get (8)

$2y^2 - 5y + 3 = 0$

$\therefore (2y-3)(y-1) = 0$

$\therefore y = \frac{3}{2}$, $y = 1$

$\therefore$ The pair of lines intersects Y-axis at $C(0,1)$ and $D(0, 3/2)$

Let $x^2 + y^2 + 2gx + 2fy + c = 0$ --- (1)

be the circle passing through $A(-3,0)$, $B(-\frac{1}{2}, 0)$, $C(0,1)$

$\therefore$ The co-ordinates of these points satisfy equation of that circle

$\therefore$ $9 - 6g + c = 0$ --- (2)

$\frac{1}{4} - g + c = 0$ --- (3)

$1 + 2f + c = 0$ --- (4)

Subtracting (3) from (2),

$-5g + \frac{35}{4} = 0$

$\Rightarrow g = 7/4$

Using (3), $\frac{1}{4} - \frac{7}{4} + c = 0$

$\Rightarrow c = 3/2$

Using (4), $1 + 2f + \frac{3}{2} = 0$

$\Rightarrow f = -5/4$

$\therefore$ The required circle is

$x^2 + y^2 + \frac{7}{2}x - \frac{5}{2}y + \frac{3}{2} = 0$

$\therefore 2x^2+2y^2+7x-5y+3=0$ ---- (5) ⑨

For $D(0, 3/2)$,

$0 + 2\times\frac{9}{4} + 0 - 5\times\frac{3}{2} + 3$

$= \frac{9}{2} - \frac{15}{2} + 3 = 0$

$\therefore D$ is also a point on (5)

$\therefore A, B, C, D$ are on a circle, whose eqⁿ is given by (5)

OR

(1) Let $x^2+y^2+2gx+2fy+c=0$ be the required circle.

The centre $C(-g,-f)$ of this circle is on line $3x+4y+6=0$

$\therefore -3g-4f+6=0$ ---- (1)

The required circle is orthogonal to the given circles

$\therefore$ using $2g_1g_2+2f_1f_2 = c_1+c_2$, we get

$2g(-3)+2f(0) = c+1$ ---- (2)

$2g(0)+2f(-2) = c+1$ ---- (3)

Solving (1), (2), (3) for g, f, c, we get

$g = \frac{2}{3}$, $f = 1$, $c = -5$

$\therefore$ The required circle is

$x^2+y^2+2gx+2fy+c=0$

ie $x^2+y^2+\frac{4}{3}x+2y-5=0$

ie $3x^2+3y^2+4x+6y-15=0$

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(2) Solving $x+y=7$ and

$x^2+y^2=25$, $y=7-x$

$\therefore x^2+(7-x)^2=25$

$\therefore x^2+x^2-14x+49=25$

$\therefore 2x^2-14x+24=0$

$\therefore x^2-7x+12=0$

$\therefore (x-4)(x-3)=0$

$\therefore x=4$ or $x=3$

$\therefore x=4 \Rightarrow y=7-x=7-4=3$

$\therefore x=3 \Rightarrow y=7-x=7-3=4$

$\therefore$ The points of intersection are

$A(4,3)$, $B(3,4)$.

$\therefore$ Intersection set : $\{(4,3),(3,4)\}$

Q2 (a)

Slope of $x+y=0$ is -1.

Let the slope of line making angle of measure α with $x+y=0$ be m, then

$$\tan\alpha = \left|\frac{m-(-1)}{1+m(-1)}\right| = \left|\frac{m+1}{1-m}\right|$$

$$\therefore \frac{m+1}{1-m} = \tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

or

$$\frac{m+1}{m-1} = \frac{\sin\alpha}{\cos\alpha}$$

$$\therefore \frac{m+1-(1-m)}{m+1+1-m} = \frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha} \quad \text{or}$$

(11)

$$\frac{m+1+m-1}{m+1-(m-1)} = \frac{\sin\alpha+\cos\alpha}{\sin\alpha-\cos\alpha}$$

$\therefore m = \dfrac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}$ or $m = \dfrac{\sin\alpha+\cos\alpha}{\sin\alpha-\cos\alpha}$

These two values of m are the slopes of the required pair of lines through origin. Denote them by m_1 & m_2

$$\therefore m_1+m_2 = \frac{(\sin\alpha-\cos\alpha)^2+(\sin\alpha+\cos\alpha)^2}{\sin^2\alpha-\cos^2\alpha}$$

$$= -\frac{2}{\cos 2\alpha}$$

$\therefore m_1+m_2 = -2\sec 2\alpha$ and $m_1 m_2 = 1$

$\therefore$ The equation of pair of lines is

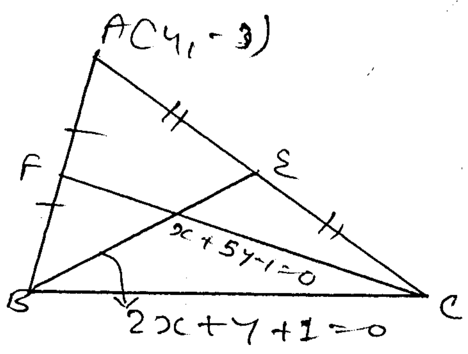
$$y^2-(m_1+m_2)xy+m_1m_2x^2 = 0$$

$$\therefore y^2-(-2\sec 2\alpha)xy+(1)x^2 = 0$$

$$\therefore y^2+2xy\sec 2\alpha+x^2 = 0$$

OR

(D)



Point A (4, −3) is not on the given lines containing medians ((4, −3), does not satisfy any of the equations $2x+y+1=0$ and $x+5y-1=0$)

$\therefore$ Take point A and the lines along the medians as shown in the figure

(12)

Let $B = B(a, b)$, then B is on $\overleftrightarrow{BE} : 2x + y + 1 = 0$

ie $2a + b + 1 = 0$ ------- (1)

mid-point of $\overline{AB}$ ie. F

$= F\left(\frac{a+4}{2}, \frac{b-3}{2}\right)$ is on $\overleftrightarrow{CF} : x + 5y - 1 = 0$

$\therefore \frac{a+4}{2} + \frac{5(b-3)}{2} - 1 = 0$

$\therefore a + 5b - 13 = 0$ ----- (2)

Solving (1) and (2), we get

$a = -2, \ b = 3$

$\therefore B(a, b) = B(-2, 3)$

Let $C = C(c, d)$, C is on $\overleftrightarrow{CF}$:

$x + 5y - 1 = 0$

$\therefore c + 5d - 1 = 0$ ----- (3)

The mid point of $\overline{AC}$:

ie $E = E\left(\frac{4+c}{2}, \frac{d-3}{2}\right)$,

E is on $\overleftrightarrow{BE} : 2x + y + 1 = 0$

$\therefore \frac{2(4+c)}{2} + \frac{d-3}{2} + 1 = 0$

$\therefore 2c + d + 7 = 0$ ---- (4)

Solving (3) and (4), we get

$d = 1, \ c = -4$

$\therefore C(c, d) = C(-4, 1)$

(13)

Q 3

Ans : 3 (A)

(1) Theorem Ch : 5 - Parabola

(2) Let the end points of focal chord $\overline{PQ}$ be $P(t_1)$ and $Q(t_2)$ and let $\overleftrightarrow{PQ}$ form an angle of measure 60° with positive x - axis

$\therefore t_1 t_2 = -1$ and

$\tan\theta$ = slope of $\overleftrightarrow{PQ} = \dfrac{2at_1 - 2at_2}{at_1^2 - at_2^2}$

$$= \frac{2}{t_1 + t_2}$$

$\therefore t_1 + t_2 = 2\cot\theta$ and $t_1 t_2 = -1$

$$\therefore PQ^2 = (at_1^2 - at_2^2)^2 + (2at_1 - 2at_2)^2$$
$$= a^2(t_1^2 - t_2^2)^2 + 4a^2(t_1 - t_2)^2$$
$$= a^2(t_1^2 - t_2^2)\left[(t_1 + t_2)^2 + 4\right]$$
$$= a^2\left[(t_1 + t_2)^2 - 4t_1 t_2\right]\left[(t_1 + t_2)^2 + 4\right]$$
$$= a^2\left[(t_1 + t_2)^2 + 4\right]^2 \quad (\because t_1 t_2 = -1)$$
$$= a^2(4\cot^2\theta + 4)^2 = a^2(4\csc^2\theta)^2$$

$$PQ = 4a\csc^2\theta$$

OR

(14)

Let $P(x_1, y_1)$ be the point in the plane of the parabola $y^2 = 4ax$

Let m be the slope of tangent which passes through $P(x_1, y_1)$

$\therefore y = mx + \frac{a}{m}$ is the equation of such tangent

$\therefore y_1 = mx_1 + \frac{a}{m}$

$\therefore m$ satisfies $x_1 m^2 - y_1 m + a = 0$ ---- (1)

$\therefore$ If $\Delta = y_1^2 - 4ax_1 > 0$, then there are two roots m_1 and m_2 of equation (1)

$\therefore m_1 + m_2 = \frac{y_1}{x_1}$, $m_1 m_2 = \frac{a}{x_1}$

If sum of the slopes of tangent through $P(x_1, y_1)$ is constant, then

$\frac{y_1}{x_1} = m_1 + m_2 = k$ (Constant)

$\therefore (x_1, y_1)$ satisfies $y = kx$ which is the required equation of the set of points in (1)

If the product of the slopes of tangent is constant, then

$\frac{a}{x_1} = m_1 \cdot m_2 = k$

$\therefore kx_1 = a$

$\therefore (x_1, y_1)$ satisfies $kx = a$, which is the eq^n of set of points in (2)

(15)

Q 3 (B)

(1) Theorem Ch : 6 Ellipse

(2) $P(\alpha)$ and $Q(\beta)$ are the points on ellipse such that $\overline{PQ}$ passes through $S(ae, 0)$

The equation of $\overleftrightarrow{PQ}$ is

$$\frac{x}{a}\cos\frac{\alpha+\beta}{2} + \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha-\beta}{2}$$

The line passes through $s(ae, 0)$

$$\frac{ae}{a}\cos\frac{\alpha+\beta}{2} + 0 = \cos\frac{\alpha-\beta}{2}$$

$$\therefore e = \frac{\cos\frac{\alpha-\beta}{2}}{\cos\frac{\alpha+\beta}{2}}$$

$$\therefore \frac{e-1}{e+1} = \frac{\cos\frac{\alpha-\beta}{2} - \cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2} + \cos\frac{\alpha+\beta}{2}}$$

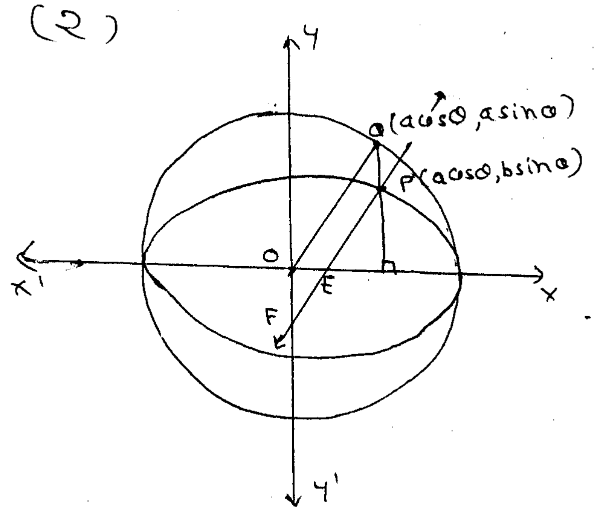
$$= \frac{2\sin\alpha/2\ \sin\beta/2}{2\cos\alpha/2\ \cos\beta/2}$$

$$= \tan\alpha/2\ \tan\beta/2$$

OR

(16)

(2)



Let $P(a\cos\theta, b\sin\theta)$ $(-\pi < \theta \leq \pi)$ be a point on ellipse $(\theta \neq 0)$

$\therefore$ The point corresponding to P on the auxiliary circle is $Q(a\cos\theta, a\sin\theta)$

$\therefore$ The slope of $\overleftrightarrow{OQ}$

$$= \frac{a\sin\theta - 0}{a\cos\theta - 0} = \tan\theta$$

$\therefore$ Equation of line parallel to $\overleftrightarrow{OQ}$ through P is

$$y - b\sin\theta = \tan\theta(x - a\cos\theta)$$

i.e. $y\cos\theta - b\sin\theta\cos\theta = x\sin\theta - a\sin\theta\cos\theta$

i.e. $x\sin\theta - y\cos\theta = (a-b)\sin\theta\cos\theta$ --- (1)

By substituting $y=0$ in (1), we get the co-ordinates of E, the intersection of line (1) with x-axis and substituting $x=0$ in (1), we get co-ord of point of intersection F of line (1) with y-axis

$\therefore E = E((a-b)\cos\theta, 0)$, $F = F(0, -(a-b)\sin\theta)$

$\therefore PE^2 = (a\cos\theta - (a-b)\cos\theta)^2 + (b\sin\theta - 0)^2$

$= b^2\cos^2\theta + b^2\sin^2\theta = b^2$ $\quad\therefore PE = b$

$PF^2 = (a\cos\theta - 0)^2 + (b\sin\theta + (a-b)\cos\theta)^2$

$= a^2\cos^2\theta + a^2\sin^2\theta = a^2$ $\quad\therefore PF = a$

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Q-3(c)(1) theorem Ch:7

(2) let $P(\theta)$ be a point on hyperbola
$P(\theta) = P(a\sec\theta, b\tan\theta)$, $C(0,0)$ is the centre of the hyperbola.

$\therefore CP^2 = a^2\sec^2\theta + b^2\tan^2\theta \quad \ldots (1)$

$S(ae, 0)$ and $S'(-ae, 0)$ are foci of the ellipse

$SP^2 = (a\sec\theta - ae)^2 + (b\tan\theta - 0)^2$

$= a^2(\sec^2\theta - 2e\sec\theta + e^2) + a^2(e^2-1)\tan^2\theta$

$= a^2\{\sec^2\theta - 2e\sec\theta + e^2 + e^2\tan^2\theta - \tan^2\theta\}$

$= a^2\{1 - 2e\sec\theta + e^2\sec^2\theta\}$ | $e > 1, |\sec\theta| \geq 1$

$= a^2(e\sec\theta - 1)^2$ | $\therefore |e\sec\theta| \geq 1$

$SP = |a(e\sec\theta - 1)|$

Similarly, $S'P = |a(e\sec\theta + 1)|$

$\therefore SP \cdot S'P = a^2(e^2\sec^2\theta - 1) = a^2\left\{\frac{a^2+b^2}{a^2}\sec^2\theta - 1\right\}$

$= \frac{a^2(a^2\sec^2\theta + b^2\sec^2\theta - a^2)}{a^2}$

$= a^2\sec^2\theta + b^2\sec^2\theta - a^2$

$= a^2\sec^2\theta + b^2(1+\tan^2\theta) - a^2$

$= a^2\sec^2\theta + b^2\tan^2\theta + b^2 - a^2$

$= CP^2 + b^2 - a^2$

$= CP^2 - a^2 + b^2$ (using (1))

(18)

Q. 3(D) (1)

Focus $S=(3,0)$, $b=2$

$\therefore (ae,0)=(3,0)$ $\therefore ae=3$

$b^2=a^2(e^2-1)=a^2e^2-a^2=(ae)^2-a^2$

$\therefore 4=9-a^2$ $\therefore a^2=9-4=5$

$\therefore$ Eq^n of curve : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\therefore \frac{x^2}{5}-\frac{y^2}{4}=1$

$4x^2-5y^2=20$

(2) centre $(-g,-f)$ $r=\sqrt{g^2+f^2-c}$

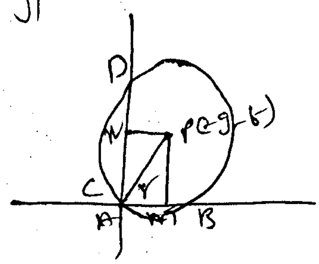
$P_1=|-f|$ $P_2=|-g|$

$AM^2=r^2-p^2$

$=g^2-c$

$AM=\sqrt{g^2-c}$

$AB=2\sqrt{g^2-c}$



CD can be obtained

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Q-4(A)

comparing the equation with

$ax^2+2hxy+by^2+2gx+2fy+c=0$, $a=b=1$

Taking $\frac{\pi}{4}$ Rotation : $x=\frac{x'-y'}{\sqrt{2}}$, $y=\frac{x'+y'}{\sqrt{2}}$

$\therefore$ Eqn in new system is

$$\frac{(x'-y')^2}{2}+\frac{2(x'-y')}{\sqrt{2}}\cdot\frac{(x'+y')}{\sqrt{2}}+\frac{(x'+y')^2}{2}+\sqrt{2}\left(\frac{x'-y'}{\sqrt{2}}\right)$$
$$-\sqrt{2}\left(\frac{x'+y'}{\sqrt{2}}\right)=0$$

$$\frac{4x'^2}{2}+x'-y'-x'-y'=0$$

$$x'^2=y'$$

$\therefore$ curve is a parabola

$\therefore$ Eccentricity is 1

comparing with $x^2=4ay$, $4a=1$ $\therefore a=\frac{1}{4}$

in (x', y') system,

Focus : $(0,a)=(0,\frac{1}{4})$; Directrix : $y'=-a=-\frac{1}{4}$

in (x', y') system,

focus: $x=\frac{x'-y'}{\sqrt{2}}=\frac{0-\frac{1}{4}}{\sqrt{2}}=-\frac{1}{4\sqrt{2}}$

$y=\frac{x'+y'}{\sqrt{2}}=\frac{0+\frac{1}{4}}{\sqrt{2}}=\frac{1}{4\sqrt{2}}$ $\therefore$ Focus $\left(\frac{-1}{4\sqrt{2}},\frac{1}{4\sqrt{2}}\right)$

(20)

Directrix : $y' = \frac{y-x}{\sqrt{2}} = \frac{-1}{4}$

$\therefore x - y = \frac{\sqrt{2}}{4}$

$\therefore x - y = \frac{1}{2\sqrt{2}}$.

OR

(A) (1) Given eqⁿ is $x^2+y^2-4x-6y-2=0$

$\therefore x^2-4x+4+y^2-6y+9-15=0$

$\therefore (x-2)^2+(y-3)^2=15$

Shift origin at $O'(2,3)$, so that $(x,y) = (X+2, Y+3)$

$\therefore$ the eqⁿ is $X^2+Y^2=15$ which is a circle.

(2) $xy=16$

$a=b=0$ $\therefore$ Taking $\frac{\pi}{4}$ Rotation axes,

$x = \frac{x'-y'}{\sqrt{2}}$, $y = \frac{x'+y'}{\sqrt{2}}$

$xy=16 \Rightarrow \frac{x'-y'}{\sqrt{2}} \cdot \frac{x'+y'}{\sqrt{2}} = 16$

$\therefore x'^2-y'^2=32$, Comparing with $x^2-y^2=a^2$

$a^2=32$ $\therefore a=\sqrt{32}=4\sqrt{2}$

$\therefore$ curve is a rectangular hyperbola.

$\therefore e=\sqrt{2}$

(21)

Q.4(B) (1) Theo. Ch : 9

(2) If the given directions are same or opposite then for some $k \in R - \{0\}$, $(2, 3, 5) = k(a, a+1, a+2)$

$\therefore ka = 2,\ k(a+1) = 3,\ k(a+2) = 5$

$\therefore ka = 2,\ ka + k = 3,\ ka + 2k = 5$

$\therefore 2 + k = 3 \ ; 2 + 2k = 5$

$k = 1 \ ; 2k = 3 \ \therefore k = \frac{3}{2}$

Thus the eqn $ka = 2,\ k(a+1) = 3,\ k(a+2) = 5$ are not consistant.

$\therefore (2, 3, 5) \neq k(a, a+1, a+2)$ for any $k \in R - \{0\}$

$\therefore$ For any $a \in R$, the given directions cannot be same or opposite.

Q.4(C) (1) Theo Ch-10.

(2) Let $P = P(\bar{\theta})$, $A = A(\bar{x})$, $B = B(\bar{y})$ and $C = C(\bar{z})$, where A, B, C are vertices of $\triangle ABC$ and P is any point. The centroid of $\triangle ABC$ is $G(\bar{g})$ where $\bar{g} = \frac{1}{3}(\bar{x} + \bar{y} + \bar{z})$

$\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$

$= \bar{x} - \bar{\theta} + \bar{y} - \bar{\theta} + \bar{z} - \bar{\theta} = \bar{x} + \bar{y} + \bar{z}$

$= 3\frac{(\bar{x} + \bar{y} + \bar{z})}{3} = 3\bar{g} = 3\overrightarrow{PG}$

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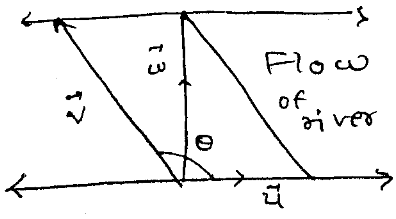
Q 4 (D)

(1)



Let $\hat{i}$ be the unit vector in the direction of flow of river and $\hat{j}$ be the unit vector $\perp^{lar}$ to the flow

Let the direction of swimmer make angle of measure θ with the direction of flow of river, so that resultant speed of swimmer be $\perp^{lar}$ to the flow

Speed of river $\vec{u} = 5\hat{i}$

Speed of swimmer is $\vec{v} = 8\cos\theta\,\hat{i} + 8\sin\theta\,\hat{j}$

The resultant speed of the swimmer is

$\vec{w} = \bar{u} + \bar{v} = 5\hat{i} + 8\cos\theta\,\hat{i} + 8\sin\theta\,\hat{j}$

$= (5 + 8\cos\theta)\,\hat{i} + 8\sin\theta\,\hat{j}$

Since $\vec{w} \perp \hat{i}$, $\vec{w}\cdot\hat{i} = 0$

$\therefore [(5 + 8\cos\theta)\hat{i} + (8\sin\theta)\hat{j}]\cdot\hat{i} = 0$

$\therefore 5 + 8\cos\theta = 0$

$\therefore \cos\theta = -5/8$

$\therefore \theta = \cos^{-1}\left(-\frac{5}{8}\right)$

$= \pi - \cos^{-1}\frac{5}{8}$

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(1) Let $\bar{x} = (1, 3)$

Suppose $\bar{y} = (y_1, y_2) \in R^2$ such that

$\bar{y} \perp \bar{x}$ and $|\bar{y}| = 1$

$\therefore \bar{y} \cdot \bar{x} = 0 \Rightarrow y_1 + 3y_2 = 0$

$\Rightarrow y_1 = -3y_2 = k$ (say)

$\therefore y_1 = k$ and $y_2 = -k/3$

$|\bar{y}| = 1 \Rightarrow k^2 + \frac{k^2}{9} = 1$

$\Rightarrow k = \pm \frac{1}{\sqrt{10}}$

$\therefore \bar{y} = \left(\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}} \right)$ or

$\bar{y} = \left(-\frac{1}{\sqrt{10}}, \frac{1}{\sqrt{10}} \right)$

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Q 5 (A)

(1) Theo. Ch - 11

(2) Theory

OR

Theory

(B) (1)

(2) Let line $L_1 : \frac{x-3}{1} = \frac{y+2}{-1} = \frac{z-1}{1} = k$

$(k \in R)$

$\therefore x-3 = k$, $y+2 = -k$, $z-1 = k$, $k \in R$

$\therefore$ Any point $P \in L_1$ Can be expressed as

$P(x,y,z) = P(3+k, -k-2, 1+k)$, $k \in R$

Let line $L_2 : \frac{x}{2} = \frac{z+3}{3}$, $y+1 = 0$

If $L_1 \cap L_2 = \{P\}$, then P is on the line L_1 and also on L_2

$\therefore$ For some $k \in R$, $(3+k, -k-2, 1+k)$ should satisfy the eq^n of L_2

$\therefore \frac{3+k}{2} = \frac{1+k+3}{3}$ and $-k-2+1 = 0$

From $-k-2+1 = 0$, $k = -1$ which also satisfies

$\frac{3+k}{2} = \frac{1+k+3}{3}$

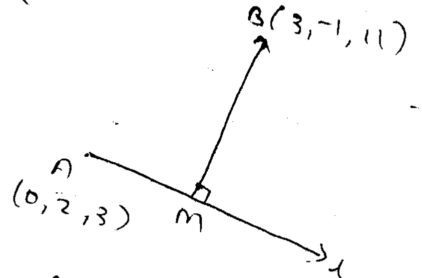
∴ The position vector (co-ordinates) of the point of intersection of L_1 and L_2 is

$(3+k, -k-2, 1+k) = (2, -1, 0)$ (For $k=-1$)

$\therefore L_1 \cap L_2 = \{P(2,-1,0)\}$

OR

(2)



Let $L : \frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4} = k$, say

∴ The direction of L is $\bar{l} = (2,3,4)$ and position vector of any point on l can be written as $(x,y,z) = (2k, 3k+2, 4k+3)$

$B(3,-1,11)$ is a point given outside the line. Let M the foot of perpendicular from B on L. M is on L ∴ for some $k \in R$, the position vector of M can be taken ~~(2k, 3k+3, 4k+8)~~ $(2k, 3k+2, 4k+3)$.

$\therefore \overrightarrow{BM} = (2k, 3k+2, 4k+3) - (3,-1,11)$
$= [(2k-3), (3k+3), (4k-8)]$

Since $\overrightarrow{BM} \perp \bar{l}$, $\overrightarrow{BM} \cdot \bar{l} = 0$

$\therefore 2(2k-3) + 3(3k+3) + 4(4k-8) = 0$

$29k - 29 = 0 \quad \therefore k = 1$

∴ The position vector of M is

$(2k, 3k+2, 4k+3) = (2,5,7) = \bar{c}$, say

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$\therefore$ Eq^n of $\overrightarrow{BM}$ can be taken as $\bar{r} = \bar{b} + k(\bar{b} - \bar{c})$
where $B(\bar{b}) = B(3, -1, 11)$ & $M(\bar{c}) = M(2, 5, 7)$
$\therefore \bar{r} = (3, -1, 11) + k[(3, -1, 11) - (2, 5, 7)]$
$\bar{r} = (3, -1, 11) + k(1, -6, 4)$, $k \in R$

This is required line $\overrightarrow{BM}$ which passes through B & which is $\perp$ar to L.

Q.5(c)(i)

$A(\bar{x}) = A(a, 0, 0)$
$B(\bar{y}) = B(0, b, 0)$
$C(\bar{z}) = C(0, 0, c)$ &
$D(\bar{w}) = D(0, 0, 0)$
let $P(\bar{u}) = P(x, y, z)$
be equal distance from A, B, C, D.

$\therefore AP^2 = (x-a)^2 + y^2 + z^2$
$BP^2 = x^2 + (y-b)^2 + z^2$
$CP^2 = x^2 + y^2 + (z-c)^2$
$DP^2 = x^2 + y^2 + z^2$

Now $AP = BP = CP = DP$
$\therefore AP^2 = BP^2 = CP^2 = DP^2$
$\therefore AP^2 = BP^2 \Rightarrow (x-a)^2 + y^2 + z^2 = x^2 + y^2 + z^2$
$\Rightarrow x^2 - 2ax + a^2 + y^2 + z^2 - x^2 - y^2 - z^2 = 0$
$\Rightarrow a^2 - 2ax = 0 \quad \therefore x = \frac{a^2}{2a} = \frac{a}{2}$

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$BP^2 = DP^2 \Rightarrow -2by + b^2 = 0$

$\therefore y = \frac{b}{2}$

$CP^2 = DP^2 \Rightarrow -2cz + c^2 = 0$

$\therefore z = \frac{c}{2}$

$\therefore (x, y, z) = \left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$

$\therefore P\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$ is equidistant from given four points

(2)

(3) $3x^2 + 3y^2 + 3z^2 - 3x - 2y - 6z - 66 = 0$

$\therefore x^2 + y^2 + z^2 - x - \frac{2}{3}y - 2z - 22 = 0$

$\therefore u = -\frac{1}{2}, \; v = -\frac{1}{3}, \; w = -1, \; d = -22$

$\therefore r^2 = u^2 + v^2 + w^2 - d$

$= \frac{1}{4} + \frac{1}{9} + 1 + 22$

$= \frac{9 + 4 + 36 + 792}{36} = \frac{841}{36} > 0$

$\therefore$ Eq^n represents a sphere

$\therefore$ Centre $C = C(-u, -v, -w) = C\left(\frac{1}{2}, \frac{1}{3}, 1\right)$

Radius : $r = \sqrt{\frac{841}{36}} = \frac{29}{6}$

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(4) $5x^2+5y^2+5z^2-5x-10y+15z+21=0$

$\therefore x^2+y^2+z^2-x-2y+3z+\frac{21}{5}=0$

$\therefore u=\frac{-1}{2}$, $v=-1$, $w=\frac{3}{2}$, $d=\frac{21}{5}$

$r^2=u^2+v^2+w^2-d$

$=\frac{1}{4}+1+\frac{9}{4}-\frac{21}{5}$

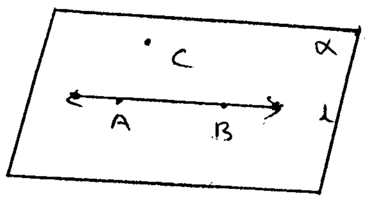
$=\frac{5+20+45-84}{20}=\frac{-19}{20}<0$

$\therefore$ Eq^n does not represent a sphere in R^3.

Q5 (D)

(1) Let us select two different points on line

$\bar{r}=(1,1,1)+k(2,1,2)$, $k\in R$



For $k=0$ and 1, we get points $A(1,1,1)$, $B(3,2,3)$ on the given lines. Point $C(1,-1,2)$ is not on given line

$\therefore$ The required plane passing through the given line and point C is same as the plane determined by non-collinear points A, B and C

∴ Its eq^n is (29)

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

ie $\begin{vmatrix} x-1 & y-1 & z-1 \\ 3-1 & 2-1 & 3-1 \\ 1-1 & -1-1 & 2-1 \end{vmatrix} = 0$

ie $\begin{vmatrix} x-1 & y-1 & z-1 \\ 2 & 1 & 2 \\ 0 & -2 & 1 \end{vmatrix} = 0$

ie $(x-1)(5) - (y-1)(2) + (z-1)(-4) = 0$

ie $5x - 2y - 4z - 5 + 2 + 4 = 0$

ie. $5x - 2y - 4z + 1 = 0$

OR

(ii) The normal to the plane $2x-3y+4z = 44$ is $(2,-3,4)$

∴ The direction of line $\perp^{lar}$ to the plane is $(2,-3,4)$ and that line passes through $A(2,-1,2)$

∴ The equation of this line is $\bar{r} = \bar{a} + k\bar{l}$, $k \in R$

where $\bar{a} = (2,-1,2)$

$\bar{l} = (2,-3,4)$

∴ $\bar{r} = (2,-1,2) + k(2,-3,4)$, $k \in R$

∴ $(x,y,z) = (2+2k, -1-3k, 2+4k)$

This is reqd eq^n of line (1)

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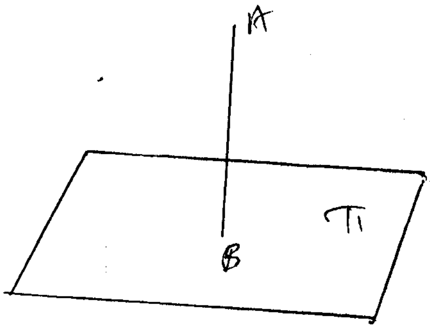

The intersection of this line with the given plane is foot of $\perp^{lar}$ from A on the plane

Let $P(x, y, z)$ be any general point on line (1), then for some $k \in R$, $P(x, y, z) = P(2+2k, -1-3k, 2+4k)$ must satisfy the eq^n of plane

$2x - 3y + 4z = 44$

$\therefore 2(2+2k) - 3(-1-3k) + 4(2+4k) = 44$

$\therefore 4 + 4k + 3 + 9k + 8 + 16k = 44$

$\therefore 29k = 29$

$\therefore k = 1$

$\therefore B = B(2+2k, -1-3k, 2+4k)$

$= B(4, -4, 6)$ (for $k = 1$)

$\therefore$ The position vector of foot of the normal is $(4, -4, 6)$

The length of the perpendicular

$AB = \sqrt{(2-4)^2 + (-1+4)^2 + (2-6)^2}$

$= \sqrt{29}$