Q. Paper set No. 3

1

MANINAGAR

TIME: 3 Hrs MATHS - I (050) MAX MARKS - 75

XII - SCI

1A 1) Obtain incentre of a triangle. (3)

2) If A is (2,3) and B is (0,7). In what ratio does the X-axis divides $\overline{AB}$ from A. (1)

B [B] Answer any two: (4)

1) If A, B, C, P are distinct and non collinear point of the plane then prove that Area of ΔPAB + Area of ΔPBC + Area of $\Delta PCA \geq$ Area of ΔABC

2) Find point C on $\overleftrightarrow{AB}$ such that 3AC = AB where A(0,1) B(2,9)

3) If (3,2) (4,5) and (2,3) are three of the four vertices of a parallelogram. What is co-ordinate of fourth vertices

C [C] Attempt any two:- (4)

1) If A (3,2) B(5,6) $\in R^2$ and $P(x,y) \in \overline{AB}$ then P. that $17 \leq 3x + 4y \leq 42$

2) Find the equation of line which passes through (3,4) and which makes an angle of $\pi/4$ with the line $3x + 4y = 2$.

3 Prove that the points (3,4) and (-2,1) [2]
are on opposite side of the line $3x-y+6=0$

D] Graph of linear equation represents a straight line (3)

2 A] Prove that if $ax^2+2hxy+by^2+2gx+2fy+c=0$ (3)
represents a pair of lines then this pair
is parallel to the pair of lines represented
by the pair of lines represented by $ax^2+2hxy+by^2=0$ where $a^2+b^2+h^2\neq0$

2. Find the angle between the lines (1)
represented by $x^2-2xy\sec\alpha+y^2=0$ $0<\alpha<\pi/2$

B] (1) Find the condition for the line (2)
$y=mx+c$ be tangent to the circle
$x^2+y^2=r^2$ and the point of contact.

2) The sides of a triangle are along the lines $2x-3y+5=0$ and $3x+2y+7=0$. Find ortho centre and $x=2$ (2)

3) If $px^2+3y^2+(q-3)xy+2px+3qy-3=0$ (1)
represent a circle then find centre and radius

C(1)] Find the equation of the circle passing through the points (5, -8) (-2, 9) and (2, 1) (3) (3)

[OR]

Find the equation of tangents to the circle $x^2+y^2=17$ from the point (5, 3)

2) For $\lambda \in R-\{a\}$ show that line $\frac{x}{a-\lambda}+\frac{y}{b}=1$ (1) passes through a fixed point

D). Find the area of the parallelogram whose sides are along the line $y=mx+a$, $y=mx+b$, $y=nx+c$ and $y=nx+d$ (3)

[OR]

Prove that if $a+b+c=0$ and $b^2 \neq ac$, $c^2 \neq ab$ and $a^2 \neq bc$, then the lines $ax+by+c=0$ $bx+cy+a=0$ and $cx+ay+b=0$ are concurrent and find the point of concurrence.

3, A] (1) Obtain standard equation of parabola (2)

(2) If the focus of the parabola $y^2=4ax$ divides a focal chord in the ratio 1 : 2 then find the equation of the line containing the focal chord. (2)

[OR]

Show that the line $3y=6x+2$ touches the parabola $3y^2=16x$. Find the point of contact.

B] (1) Obtain the equation of the tangent at the point (x_1, y_1) of the ellipse and hence obtain the equation of the tangent at 'θ' point of the ellipse (2) (4)

2) If the difference of the eccentric angles of P and Q is $\pi/2$ and O is the origin then prove that the area of ΔPOQ is $\frac{1}{2}ab$ for ellipse (2)

[OR]

The tangent at the point P intersect a directrix at F. Prove that PF forms a right angle at the corresponding focus

C] (1). Define rectangular hyperbola. (2) Obtain its standard equation and eccentricity

(2) Show that the angle between two asymptotes of the hyperbola $x^2 - 2y = 1$ is $\tan^{-1} 2\sqrt{2}$ (2).

D] (1). If S (4,0) and $e = 3/2$ find the equation of hyperbola. (1)

2) Find the set of all points P outside a circle, such that the tangents drawn to a circle from P are ⊥lar to each other (2)

5

4) A] which curve is represented by the equation $3x^2+8xy-3y^2-20x+10y-15=0$ find the coordinates of focci, equations of directrix, and eccentricities (4)

OR

Identify the following curves by obtaining their standard form :

1) $x^2+y^2-4x-6y-2=0$

2) $x^2-y^2+4x+2y+3=0$

(2)

B] (1) Obtain necessary and sufficent condition for two vectors $\bar{x}=(x_1,x_2)$ $\bar{y}=(y_1,y_2)$ to be collinear $(\bar{x},\bar{y}\neq\theta)$

(2)

2) If $\bar{x},\bar{y},\bar{z}$ are non collinear, prove that $\bar{x}+\bar{y}$, $\bar{y}+\bar{z}$, $\bar{z}+\bar{x}$ are also non collinear.

(2)

C] (1) Obtain formula for that volume of prism

(2)

2) If A-P-B and if $\frac{AP}{PB}=\frac{m}{n}$ then, for any point O in space prove that $n\,\overline{OA}+m\,\overline{OB}=(m+n)\,\overline{OP}$

[6]

[D] (1) A boat speeds in the north at $6\sqrt{2}$ kms. A man on the boat feels that the wind is blowing from the south-east at 5 kms. Find the true velocity of the wind. (2)

2) Find a, if $(2a\bar{i} + \bar{j} - 4\bar{k}) \perp (a\bar{i} - 2\bar{j} + \bar{k})$ (1)

5[A] (1) In usual notations obtain the distance between given point and given line in R^3 (2)

2) Obtain equation of a plane passing through two intersecting lines

[OR] (2)

Obtain equation of plane passing through two parallel lines.

[B] (1) Find vector and cartesian equation of a sphere having centre $C(\bar{c})$ and radius r. (1)

2) If the direction cosines l, m, n of the two lines satisfy $l+m+n=0$ and $l^2-m^2+n^2=0$ show that the angle between the two lines is $\pi/2$ (3)

[OR]

Obtain shortest distance between the lines [7]

$\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$ and $\frac{x+1}{2} = \frac{y-1}{1} = \frac{9-2}{3}$

C] (1) Show that (4, 5, 1) (0, -1, -1), (3, 9, 4) (-4, 4, 4) cannot be vertices of any tetrahedron. (2)

2) obtain the equation, the centre and radius of the sphere through (0, 0, 0) (a, 0, 0), (0, b, 0) & (0, 0, c) (2)

D) Obtain the equation of plane passing through $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ and (3)

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$

[OR]

Obtain the intersection of the plane $x+y+2z=4$ and $2x-y+z+1=0$.

Solution of paper set No. 3 1]
Mathematics - I (050)(E)

1 A

(1) Text page – 19

(2) Here, A is (2, 3) and B is (0, 7)

Supp. the pt. P(x, 0) of the X-axis divides $\overline{AB}$ from A in ratio m:n ; where $m+n \neq 0$

$\therefore$ according to the y-coordinate,

$$y = \frac{my_2 + ny_1}{m+n} \text{ of P,}$$

$$0 = \frac{m(7) + n(3)}{m+n}$$

$\therefore 7m + 3n = 0$

$\therefore 7m = -3n$

$\therefore \frac{m}{n} = \frac{-3}{7}$

$\therefore m : n = -3:7$

$\therefore$ the x-axis divides $\overline{AB}$ from A at point P(x, 0) in the ratio m:n = -3:7

1 B

(1) Supp. $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and P(0,0) are the distinct noncollinear pts. of a plane

Here, for ΔABC, $D = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

and if the determine corresponding to ΔPAB, ΔPBC and ΔPCA are D_1, D_2 and D_3 resp., then

$D_1 + D_2 + D_3$

$$= \begin{vmatrix} 0 & 0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix}$$

$= (x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)$

$= x_1y_2 - x_1y_3 - x_2y_1 + x_3y_1 + x_2y_3 - x_3y_2$

$= x_1(y_2 - y_3) - y_1(x_2 - x_3) + 1(x_2y_3 - x_3y_2)$

$$= \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = D$$

2]

Thus $D = D_1 + D_2 + D_3$ is obtained

$\therefore |D| = |D_1 + D_2 + D_3|$

$\therefore |D| \leq |D_1| + |D_2| + |D_3|$

$\therefore \frac{1}{2}[|D_1| + |D_2| + |D_3|] \geq \frac{1}{2}|D|$

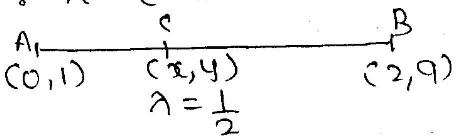
$\therefore$ the area of ΔPAB + the area of ΔPBC + area of $\Delta PCA \geq$ the area of ΔABC

(2) Here $A(0,1)$ and $B(2,9)$ and supp. C is (x,y)

Now, A, B and C are collinear and $AB = 3AC$

$\therefore$ there are two possibilities

<u>Case-1</u> : $A-C-B$



Here, if $AC = x$ then $BC = 2x$

$\therefore$ the division ratio $\lambda = \frac{AC}{BC} = \frac{x}{2x} = \frac{1}{2}$

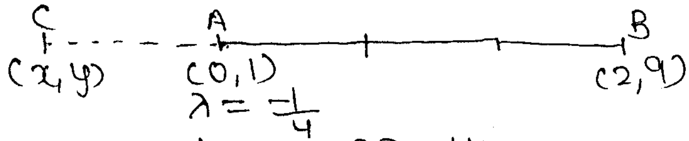
Thus, if $A-C-B$ then C divides $\overline{AB}$ from A in the ratio $1:2$

$\therefore$ using $x = \frac{mx_2 + nx_1}{m+n}$ and $y = \frac{my_2 + ny_1}{m+n}$ the coordinates of C are $x = \frac{1(2)+2(0)}{1+2} = \frac{2}{3}$

and $y = \frac{1(9)+2(1)}{1+2} = \frac{11}{3}$

$\therefore$ the coordinates of C are $\left(\frac{2}{3}, \frac{11}{3}\right)$

<u>Case-2</u> : $C-A-B$



Here, $AC = x$ then $CB = 4x$

$\therefore$ the division ratio $\lambda = -\frac{AC}{BC} = -\frac{x}{4x} = \frac{-1}{4}$

Thus, if $C-A-B$ then C divides $\overline{AB}$ from A in ratio $-1:4$

$\therefore$ the coordinates of C are

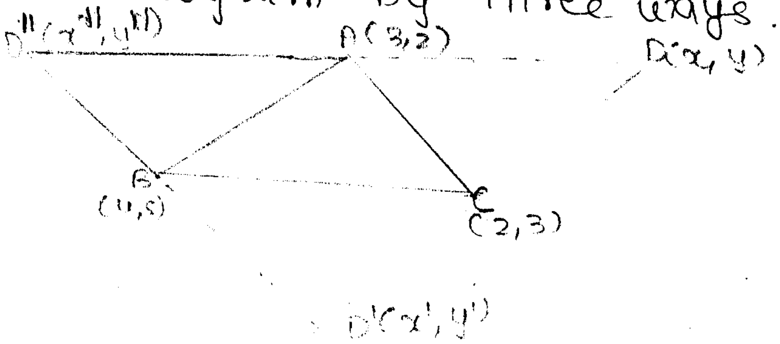
3]

$x = \frac{-1(2)+4(0)}{-1+4} = \frac{-2}{3}$ and $y = \frac{-1(9)+4(1)}{-1+4} = \frac{-5}{3}$

∴ the coordinates of C are $\left(\frac{-2}{3}, \frac{-5}{3}\right)$

Thus the coordinates of C are $\left(\frac{2}{3}, \frac{11}{3}\right)$ or $\left(\frac{-2}{3}, \frac{-5}{3}\right)$

(3) Supp. A is (3, 2), B is (4, 5) and C is (2, 3)
Here, we can get the fourth vertex of the parallelogram by three ways.



(1) □ABCD is a parallelogram and if the coord. of D are (x, y) then the midpoints of the diagonals $\overline{AC}$ and $\overline{BC}$ are same

∴ $\frac{x+4}{2} = \frac{3+2}{2}$ and $\frac{y+5}{2} = \frac{2+3}{2}$

∴ $x = 1$ and $y = 0$

∴ we get D(1, 0)

(2) □ABD'C is a parallelogram and the coordinates of D' are (x', y') then the midpts of the diagonals $\overline{AD'}$ and $\overline{BC}$ are same

∴ $\frac{x'+3}{2} = \frac{4+2}{2}$ and $\frac{y'+2}{2} = \frac{5+3}{2}$

∴ $x' = 3$ and $y' = 6$

∴ we get D'(3, 6)

(3) □ABCD'' is a parallelogram and if the coordinates of D'' are (x'', y'') then the

midpoints of the diagonals $\overline{AB}$ and $\overline{CD''}$ are same. (4)

$\therefore \frac{x''+2}{2} = \frac{3+4}{2}$ and $\frac{y''+3}{2} = \frac{2+5}{2}$

$\therefore x'' = 5$ and $y'' = 4$

$\therefore$ we get $D''(5,4)$

Thus, the fourth vertex of the given parallelogram is $(1,0)$ or $(3,6)$ or $(5,4)$

C (1) For $A(3,2)$, $B(5,6)$

Parametric eq^n of $\overleftrightarrow{AB}$

$x = tx_2 + (1-t)x_1$ $\quad$ $y = ty_2 + (1-t)y_1$

$= 5t + (1-t)3$ $\quad$ $= 6t + (1-t)2$

$= 2t + 3$ $\quad$ $= 4t + 2$

$\therefore 3x + 4y = 6t + 9 + 16t + 8$

$= 25t + 17$

But $P(x,y) \in \overline{AB}$

$\therefore 0 \leq t \leq 1$

$\therefore 0 \leq 25t \leq 25$

$\therefore 17 \leq 25t + 17 \leq 42$

$\therefore 17 \leq 3x + 4y \leq 42$

(2) Here the slope of line $3x + 4y - 2 = 0$ is $m = -\frac{3}{4}$

Supp., the slope of required line is m_2

Also, the measure of the angle bet^n these two lines is $\alpha = 45°$

Now, acc. to $\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\therefore \tan 45° = \left|\frac{-3/4 - m_2}{1 + (-3/4)m_2}\right|$

$\therefore 1 = \left|\frac{-3 - 4m_2}{4 - 3m_2}\right|$

(5)

$\therefore \dfrac{-3-4m_2}{4-3m_2} = \pm 1$

$\therefore \dfrac{-3-4m_2}{4-3m_2} = 1$ or $\dfrac{-3-4m_2}{4-3m_2} = -1$

$\therefore -3-4m_2 = 4-3m_2$ $\qquad$ $\therefore -3-4m_2 = -4+3m_2$

$\therefore -7 = m_2$ $\qquad$ $\therefore 1 = 7m_2$

$\therefore m_2 = -7$ $\qquad$ $\therefore m_2 = \dfrac{1}{7}$

$\therefore$ two lines are possible

Now, these two lines pass through pt. (3,4)

$\therefore$ their equations, acc. to $y-y_1 = m(x-x_1)$ are,

$y-4 = -7(x-3)$ or $y-4 = \dfrac{1}{7}(x-3)$

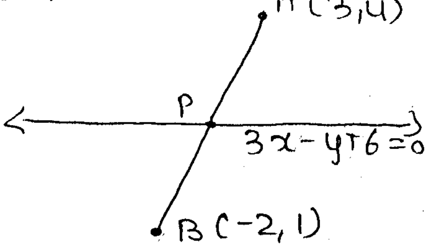
$\therefore y-4 = -7x+21$ $\qquad$ $\therefore 7y-28 = x-3$

$\therefore 7x+y-25 = 0$ $\qquad$ $\therefore x-7y+25 = 0$

Thus, the required lines are $x-7y+25=0$ and $7x+y-25=0$

(3) Supp. A is (3,4) and B is (−2,1)

Here, supp. the line $3x-y+6=0$ divides $\overline{AB}$ from A in ratio λ ($\lambda \neq 1$)



Here the coord. of the pt. P, using $\left(\dfrac{\lambda x_2+x_1}{\lambda+1}, \dfrac{\lambda y_2+y_1}{\lambda+1}\right)$ are $\left(\dfrac{\lambda(-2)+3}{\lambda+1}, \dfrac{\lambda(1)+4}{\lambda+1}\right) = \left(\dfrac{-2\lambda+3}{\lambda+1}, \dfrac{\lambda+4}{\lambda+1}\right)$

Now, P is the element of line $3x-y+6=0$

$\therefore 3\left(\dfrac{-2\lambda+3}{\lambda+1}\right) - \left(\dfrac{\lambda+4}{\lambda+1}\right) + 6 = 0$

$\therefore -6\lambda+9-\lambda-4+6\lambda+6 = 0$

$\therefore -\lambda+11 = 0$

$\therefore \lambda = 11$

6]

Here, $\lambda > 0$

$\therefore$ we get A – P – B

$\therefore$ the pts. A and B are in the opposite half planes of line $3x - y + 6 = 0$

ie the pts (3, 4) and (–2, 1) are on the opp. side of line $3x - y + 6 = 0$.

1 D Text page 36.

2 A

(1) Text Pg 63

(2) Comparing the eq^n $x^2 - 2xy\sec\alpha + y^2 = 0$ with the general quadratic equation of a pair of lines $ax^2 + 2hxy + by^2 = 0$, $a = 1$, $h = -\sec\alpha$ and $b = 1$.

Now, if the measure of the angle bet^n the lines is θ then acc. to

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{|a + b|}, \quad \tan\theta = \frac{2\sqrt{\sec^2\alpha - 1}}{1 + 1}$$

$$= \sqrt{\tan^2\alpha}$$

$$= \tan\alpha$$

$(\because \tan\alpha > 0$ for $0 < \alpha < \frac{\pi}{2})$

$\therefore \theta = K\pi + \alpha, K \in Z$

$\therefore \theta = \alpha$ $(\because \theta$ is acute angle $\therefore K = 0)$

Thus the measure of the required angle is α unit

(B)

(1) Text Pg 76

(2) Supp. lines $l_1 : 2x - 3y + 5 = 0$

$l_2 : 3x + 2y + 7 = 0$

Here slope of l_1, $m_1 = 2/3$

slope of l_2, $m_2 = -3/2$

$m_1 m_2 = -1$

(7)

$\therefore l_1 \perp l_2$

$\therefore$ solving l_1 and l_2 we get

orthocentre $(x, y) = \left(\frac{-21-10}{4+9}, \frac{-14+15}{4+9}\right)$

$= \left(\frac{-31}{13}, \frac{1}{13}\right)$

(3) eqn represents a circle

$\therefore$ coefficient of $xy = 0$

$\therefore q - 3 = 0$

$\therefore q = 3$

and coeff. of x^2 = coeff. of $y^2 = 0$

$\therefore p = 3$

$\therefore$ circle : $3x^2 + 3y^2 + 6x + 9y - 3 = 0$

$\therefore x^2 + y^2 + 2x + 3y - 1 = 0$

$\therefore g = 1, f = \frac{3}{2}, c = -1$

$\therefore$ Centre $(-g, -f) = (-1, -3/2)$

and radius $r = \sqrt{g^2 + f^2 - c}$

$= \sqrt{1 + \frac{9}{4} + 1}$

$= \frac{\sqrt{17}}{2}$ units

C

(1) Eqn of a circle whose diametrically opp. pts. are $(-2, 9)$ and $(2, 1)$ is

$(x+2)(x-2) + (y-9)(y-1) = 0$

ie $x^2 + y^2 - 10y + 5 = 0$ ... (1)

The eqn of the line passing through $(-2, 9)$ and $(2, 1)$ is

$y - 9 = \frac{1-9}{2+2}(x+2)$

ie $2x + y - 5 = 0$ ... (2).

The gen. eqn of the circle passing through the pt. of intersection of (1) & (2) is

8]

$x^2+y^2-10y+5+\lambda(2x+y-5)=0$

If this circle passes through $(5,-8)$ then

$25+64+80+5+\lambda(10-8-5)=0$

$\therefore \lambda = 58$

$\therefore$ substituting value in (3)

Required circle is

$x^2+y^2+116x+48y-285=0$

OR

Here the pt. $(5,3)$ is not on the circle $x^2+y^2=17$

So we will take the tangents to circle $x^2+y^2=r^2$ with slope m as $y=mx\pm r\sqrt{1+m^2}$

Which is passing through $(5,3)$ & $r=\sqrt{17}$

$\therefore 3=5m\pm\sqrt{17}\sqrt{1+m^2}$

$\therefore (3-5m)^2=17(1+m^2)$

$\therefore 9-30m+25m^2=17+17m^2$

$\therefore 8m^2-30m-8=0$

$\therefore 4m^2-15m-4=0$

$\therefore 4m^2-16m+m-4=0$

$\therefore (4m+1)(m-4)=0$

$\therefore m=-\frac{1}{4}$ or $m=4$

Now,

(1) taking $m=4$ and $r=\sqrt{17}$

the tangents to circle are $y=4x\pm\sqrt{17}\sqrt{1+16}$

$\therefore y=4x\pm 17$

$\therefore 4x-y\pm 17=0$

Here the pt. $(5,3)$ is not on line $4x-y+17=0$

but it is on line $4x-y-17=0$

$\therefore$ we will take tangent as $4x-y-17=0$

(2) taking $m=-\frac{1}{4}$ & $r=\sqrt{17}$

the tangents to circle are $y=-\frac{1}{4}\pm\sqrt{17}\sqrt{1+\frac{1}{16}}$

$\therefore 4y = -x \pm 17$ 9]

$\therefore x + 4y \pm 17 = 0$

Here, the pt. $(5, 3)$ is not on line $x+4y+17=0$ but it is on line $x+4y-17=0$

$\therefore$ we will take the tangent as $x+4y-17=0$

Thus, the eq^n of the tangent to the given circle from the given pt. are $4x-y-17=0$ and $x+4y-17=0$

(2) line $\frac{x}{a-\lambda} + \frac{y}{b} = 1$

$\therefore \frac{y}{b} - 1 = \frac{-x}{a-\lambda}$

$\therefore y - b = \frac{-b}{a-\lambda}(x-0)$

$\therefore y - b = m(x-0)$ where $m = -\frac{b}{a-\lambda}$

Comparing with $y - y_1 = m(x - x_1)$

Given line passes through fixed pt.

$(x_1, y_1) = (0, b)$

:D

Here solving eq^n $y = mx + a$ and $y = nx + c$,
also $y = mx + a$ and $y = nx + d$

we get $A\left(\frac{c-a}{m-n}, \frac{mc-na}{m-n}\right)$ and $B\left(\frac{d-a}{m-n}, \frac{md-na}{m-n}\right)$

$$AB^2 = \left(\frac{c-a}{m-n} - \frac{d-a}{m-n}\right)^2 + \left(\frac{mc-na}{m-n} - \frac{md-na}{m-n}\right)^2$$

$$= \left(\frac{c-d}{m-n}\right)^2 + m^2\left(\frac{c-d}{m-n}\right)^2$$

$$= \left(\frac{c-d}{m-n}\right)^2 (1+m^2)$$

$$\therefore AB = \left|\frac{c-d}{m-n}\right| \cdot \sqrt{1+m^2}$$

Also, the $\perp$r dist. bet^n $\overline{AB}$ & $\overline{CD}$ is $p_1 = \frac{|a-b|}{\sqrt{1+m^2}}$

10]

Now, the area of parallelogram $= AB \cdot p_1$

$$= \left|\frac{c-d}{m-n}\right| \cdot \sqrt{1+m^2} \cdot \frac{|a-b|}{\sqrt{1+m^2}}$$

$$= \left|\frac{(a-b)(c-d)}{m-n}\right|.$$

OR.

Here the lines are $ax+by+c=0 \cdots (1)$

$bx+cy+a=0 \cdots (2)$

& $cx+ay+b=0 \cdots (3)$

and $b^2 \neq ac$, $c^2 \neq ab$ and $a^2 \neq bc$

Now, $a_1b_2 - a_2b_1 = ca - b^2 \neq 0$

$a_2b_3 - a_3b_2 = ab - c^2 \neq 0$

and $a_3b_1 - a_1b_3 = bc - a^2 \neq 0$

and $D = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix}$$

($\because$ by applying $R_{21}(1), R_{31}(1)$)

$$= \begin{vmatrix} 0 & 0 & 0 \\ b & c & a \\ c & a & b \end{vmatrix} \quad (\because a+b+c=0)$$

$= 0$

$\therefore$ given lines are concurrent

Now, to get pt. of concurrence, solving eqn (1) and (2) using Cramer's rule

Here, $\dfrac{x}{\begin{vmatrix} b & c \\ c & a \end{vmatrix}} = \dfrac{-y}{\begin{vmatrix} a & c \\ b & a \end{vmatrix}} = \dfrac{1}{\begin{vmatrix} a & b \\ b & c \end{vmatrix}}$

$\therefore \dfrac{x}{ab-c^2} = \dfrac{-y}{-(bc-a^2)} = \dfrac{1}{ac-b^2}$

$\therefore x = \dfrac{ab-c^2}{ac-b^2}$ and $y = \dfrac{bc-a^2}{ac-b^2}$

11]

$= \frac{b(-b-c)-c^2}{c(-b-c)-b^2}$ $= \frac{b(-b-a)-a^2}{a(-a-b)-b^2}$

$= \frac{-b^2-bc-c^2}{bc-c^2-b^2}$ $= \frac{-ab-b^2-a^2}{-a^2-ab-b^2}$ ($\because a+b+c=0 \Rightarrow a=-b-c$ and $c=-a-b$)

$= 1$ $= 1$

$\therefore$ the coordinates of pt. of concurrence are $(1, 1)$

3 A

(1) Text Pg 87

(2) Supp. $\overline{PQ}$ is focal chord of parabola $y^2=4ax$ and the coord. of P and Q are $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ resp. and focus $S(a,0)$. Here, S, P and Q are collinear pts. we get $t_1 t_2 = -1$

$\therefore$ taking $t_2 = \frac{-1}{t_1}$, we get the coord. $\left(\frac{a}{t_1^2}, \frac{-2a}{t_1}\right)$ of Q

Now, $S(a,0)$ divides $\overline{PQ}$ from P in ratio 2:1

$\therefore$ acc. to $y = \frac{\lambda y_2 + y_1}{\lambda + 1}$

The y-coord of P is

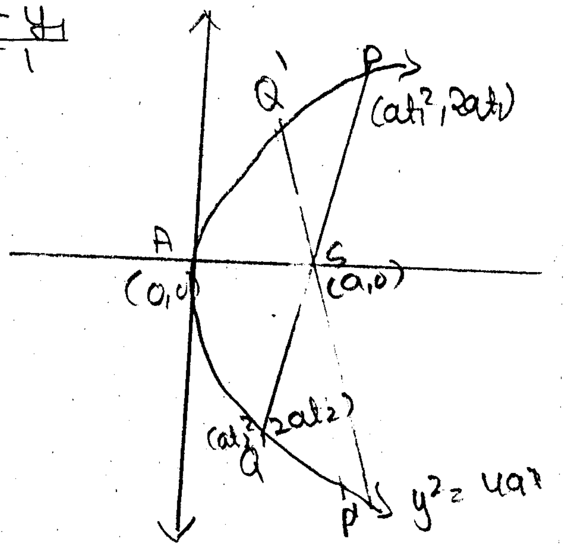
$0 = \frac{2\left(-\frac{2a}{t_1}\right) + 2at_1}{2+1}$

$\therefore \frac{4a}{t_1} = 2at_1$

$\therefore \frac{2}{t_1} = t_1$

$\therefore t_1^2 = 2$

$\therefore t_1 = \pm\sqrt{2}$



Now, 12]

(1) Taking $t_1 = \sqrt{2}$, we get coord. $(2a, 2\sqrt{2}a)$ of P

Here, the focal chord is passing through S and P

∴ eqⁿ of line containing the focal chord is,

$$\begin{vmatrix} x & y & 1 \\ a & 0 & 1 \\ 2a & 2\sqrt{2}a & 1 \end{vmatrix} = 0$$

$\therefore -2\sqrt{2}ax + ay + 2\sqrt{2}a^2 = 0$

$\therefore 2\sqrt{2}x - y - 2\sqrt{2}a = 0$

$\therefore y = 2\sqrt{2}(x-a)$

(2) Taking $t_1 = -\sqrt{2}$, we get coord. $(2a, -2\sqrt{2}a)$ of P

Hence, the second eqⁿ of line containing focal chord is $y = -2\sqrt{2}(x-a)$

Thus, the two eqⁿ of focal chord are $y = \pm 2\sqrt{2}(x-a)$

OR

Here, line $3y = 6x + 2$

$y = 2x + \frac{2}{3}$

Parabola $3y^2 = 16x$

$\therefore y^2 = \frac{16}{3}x$

$\therefore a = \frac{4}{3}$

$c = \frac{2}{3}$ & $\frac{a}{m} = \frac{4/3}{2} = \frac{2}{3}$

$\therefore c = \frac{a}{m}$

∴ given line touches given parabola.

(A) pt. of contact [3]

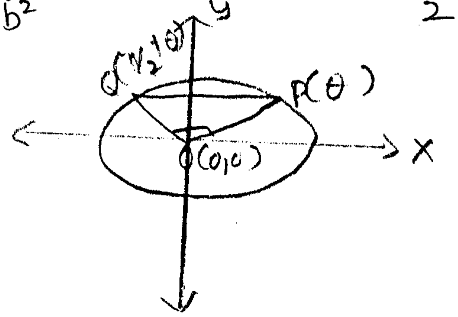
$\left(\frac{a}{m^2}, \frac{2a}{m}\right) = \left(\frac{4/3}{4}, \frac{2 \cdot 4/3}{2}\right)$

$= \left(\frac{1}{3}, \frac{4}{3}\right)$

(B)

(1) Text Pg 103

(2) Here, the diff. of eccentric angles of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ is $\frac{\pi}{2}$



∴ we will take $P(\theta)$ and $Q\left(\frac{\pi}{2}+\theta\right)$ on ellipse

Here the coord. of pt. $P(\theta)$ are $(a\cos\theta, b\sin\theta)$

and the coord. of pt. $Q\left(\frac{\pi}{2}+\theta\right)$ are

$\left(a\cos\left(\frac{\pi}{2}+\theta\right), b\sin\left(\frac{\pi}{2}+\theta\right)\right)$

ie $(-a\sin\theta, b\cos\theta)$

Also, $O(0,0)$ is the centre of ellipse

The vertices of ΔOPQ are $(0,0)$ $(a\cos\theta, b\sin\theta)$ and $(-a\sin\theta, b\cos\theta)$

$\therefore D = \begin{vmatrix} 0 & 0 & 1 \\ a\cos\theta & b\sin\theta & 1 \\ -a\sin\theta & b\cos\theta & 1 \end{vmatrix}$

$= 1\,[ab(\cos^2\theta + \sin^2\theta)]$

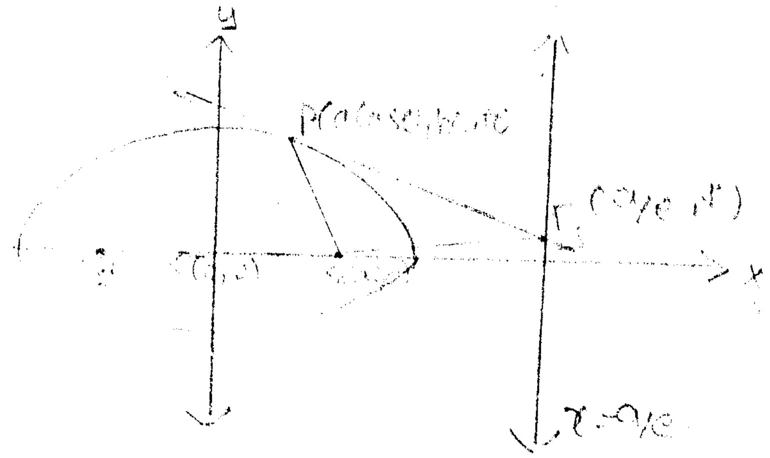
$= ab$

∴ The area of ΔOPQ $A = \frac{1}{2}|D|$ [4]

$$\Delta = \frac{1}{2}|ab| = \frac{1}{2}ab \ (\because a, b > 0)$$

Thus, the area of $\Delta OPQ = \frac{1}{2}ab$ is proved

OR.



The tangent $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ at pt. $P(a\cos\theta, b\sin\theta)$ to ellipse intersects the directrix $x = \frac{a}{e}$ of ellipse at pt. $F\left(\frac{a}{e}, k\right)$

$\therefore \frac{a}{e}\cdot\frac{\cos\theta}{a} + \frac{k}{b}\sin\theta = 1$

$\therefore \frac{k}{b}\sin\theta = 1 - \frac{\cos\theta}{e}$

$\therefore k = \frac{b(e-\cos\theta)}{e\sin\theta}$

$\therefore \left(\frac{a}{e}, \frac{b(e-\cos\theta)}{e\sin\theta}\right)$ are coord. of F & S is $(ae, 0)$

$\therefore$ the slope m_1 to $\overleftrightarrow{SF} = \dfrac{\frac{b(e-\cos\theta)}{e\sin\theta} - 0}{a/e - ae}$

$$= \frac{b(e-\cos\theta)}{\sin\theta\cdot a(1-e^2)} \quad \ldots(1)$$

and slope of m_2 of $\overleftrightarrow{SP} = \frac{b\sin\theta - 0}{a\cos\theta - ae} = \frac{-b\sin\theta}{a(e-\cos\theta)}$ ...(2)

15]

Now, from (1) and (2),

the slope $\overleftrightarrow{SF}$ · slope of $\overleftrightarrow{SP}$

$$= \frac{b(e-\cos\theta)}{\sin\theta \cdot a(1-e^2)} \cdot \left(\frac{-b\sin\theta}{a(e-\cos\theta)}\right)$$

$$= \frac{-b^2}{a^2(1-e^2)} = \frac{-b^2}{b^2} = -1$$

$\therefore \overleftrightarrow{SF} \perp \overleftrightarrow{SP}$

$\therefore \overline{PF}$ subtends a right angle at focus S.

(1) Text page 121

(2) Here the eqn of asymptotes $x^2 - 2y^2 = 0$

if the angle betn them is θ, then

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{|a+b|}$$

Here, $a = 1, h = 0, b = -2$

$$\therefore \tan\theta = \frac{2\sqrt{0-(-2)}}{|1-2|} = 2\sqrt{2}$$

$$\therefore \theta = \tan^{-1}(2\sqrt{2})$$

(2) $S(4, 0)$, $e = 3/2$

$ae = 4 \quad e = 3/2$

$a \cdot \frac{3}{2} = 4 \Rightarrow a = \frac{8}{3}$

Now, $b^2 = a^2(e^2 - 1)$

$$\therefore b^2 = \frac{64}{9}\left(\frac{9}{4} - 1\right)$$

$$= 16 - \frac{64}{9}$$

$$= \frac{144 - 64}{9}$$

$$\therefore b^2 = 80/9$$

16]

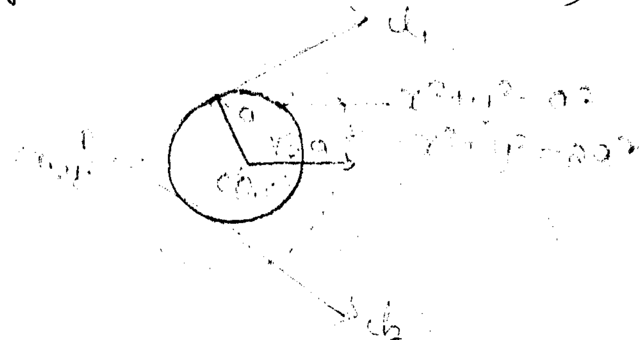
∴ Eq^n of hyperbola will be

$$\frac{9x^2}{64} - \frac{9y^2}{80} = 1.$$

(c) Supp. $y = mx \pm a\sqrt{1+m^2}$. are the tangents to the circle $x^2+y^2=a^2$ which are passing through pt. $P(x_1, y_1)$ outside the circle

∴ $y_1 = mx_1 \pm a\sqrt{1+m^2}$

∴ $(y_1 - mx_1)^2 = a^2(1+m^2)$



∴ $y_1^2 - 2x_1y_1m + m^2x_1^2 = a^2 + a^2m^2$

∴ $(a^2 - x_1^2)m^2 + 2x_1y_1m + (a^2 - y_1^2) = 0$

If m_1 & m_2 are roots of quadratic eq^n in m then $m_1m_2 = \frac{a^2-y_1^2}{a^2-x_1^2}$.

Now, tangent lines drawn from P are ⊥r to each other

∴ taking $m_1m_2 = -1$, $\frac{a^2-y_1^2}{a^2-x_1^2} = -1$

∴ $a^2 - y_1^2 = x_1^2 - a^2$

∴ $x_1^2 + y_1^2 = 2a^2$.

In general, this eq^n can be written as $x^2+y^2 = 2a^2$.

Thus locus of P is concentric circle $x^2+y^2=2a^2$ with rad. $\sqrt{2}\,a$.

17

4(A) Here, $a \neq b$

$\therefore$ On rotating the axes by an angle θ

$\tan 2\theta = \frac{2h}{a-b} = \frac{4}{3}$

$\therefore \cos 2\theta = \frac{3}{5}$

$\therefore \cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+3/5}{2}} = \sqrt{\frac{8}{10}} = \frac{2}{\sqrt{5}}$

$\therefore \sin\theta = \frac{1}{\sqrt{5}}$

Here $x = x'\cos\theta - y'\sin\theta = \frac{2x'-y'}{\sqrt{5}}$

and $y = y'\cos\theta + x'\sin\theta = \frac{x'+2y'}{\sqrt{5}}$

$\therefore$ Equation of the curve is

$3\left(\frac{2x'-y'}{\sqrt{5}}\right)^2 + 8\left(\frac{2x'-y'}{\sqrt{5}}\right)\left(\frac{x'+2y'}{\sqrt{5}}\right) - 3\left(\frac{x'+2y'}{\sqrt{5}}\right)^2$

$- 20\left(\frac{2x'-y'}{\sqrt{5}}\right) + 10\left(\frac{x'+2y'}{\sqrt{5}}\right) - 15 = 0$

$\therefore \left(\frac{12x'^2}{5} + \frac{16x'^2}{5} - \frac{3x'^2}{5}\right) + \left(\frac{3y'^2}{5} - \frac{16y'^2}{5} - \frac{12y'^2}{5}\right)$

$+ \left(\frac{-12x'y'}{\sqrt{5}} + \frac{24x'y'}{\sqrt{5}} - \frac{12x'y'}{\sqrt{5}}\right) + \left(\frac{-40x'}{\sqrt{5}} + \frac{10x'}{\sqrt{5}}\right)$

$+ \left(\frac{20y'}{\sqrt{5}} + \frac{20y'}{\sqrt{5}}\right) - 15 = 0$

$\therefore 5x'^2 - 5y'^2 - 6\sqrt{5}x' + 8\sqrt{5}y' - 15 = 0$

$\therefore 5\left(x'^2 - \frac{6x'}{\sqrt{5}} + \frac{9}{5}\right) - 5\left(y'^2 - \frac{8}{\sqrt{5}}y' + \frac{16}{5}\right) = 8$

18

$\therefore \left(x' - \frac{3}{\sqrt{5}}\right)^2 - \left(y' - \frac{4}{\sqrt{5}}\right)^2 = \frac{8}{5}$

Now on shifting origin to $\left(\frac{3}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$

$x' - \frac{3}{\sqrt{5}} = x$ and $y' - \frac{4}{\sqrt{5}} = y$

$\therefore$ The equation of the curve $X^2 + Y^2 = \left(2\frac{\sqrt{2}}{\sqrt{5}}\right)^2$, which represent the rectangular hyperbola

Here $a = 2\sqrt{\frac{2}{5}}$ $e = \sqrt{2}$

$\therefore$ the focci (x, y) system, according to $(\pm ae, 0)$ are $\left(\pm \frac{4}{\sqrt{5}}, 0\right)$ and the directrix, according to $X = \pm a/e$ $X = \pm 2/\sqrt{5}$ and

the length of both the axes $2a = 4\sqrt{\frac{2}{5}}$

Now in $(x'y')$ system the focci according to $\left(X + \frac{3}{\sqrt{5}}, y + \frac{5}{\sqrt{4}}\right)$ are

$\left(\frac{4}{\sqrt{5}} + \frac{3}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right) = \left(\frac{7}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$ and $\left(\frac{-4}{\sqrt{5}} + \frac{3}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$

$= \left(-\frac{1}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$

Now in $(x'y')$ system the focci according to $\left(\frac{2x' + 3y'}{\sqrt{5}}, \frac{y' + 2x'}{\sqrt{4}}\right)$ are $\left(\frac{2x' - y'}{\sqrt{5}}, \frac{x' + 2y'}{\sqrt{5}}\right)$ are

$\left(\frac{\frac{14}{\sqrt{5}} - \frac{4}{\sqrt{5}}}{\sqrt{5}}, \frac{\frac{7}{\sqrt{5}} + \frac{8}{\sqrt{5}}}{\sqrt{5}}\right) = (2, 3)$ and

19

$\left(\frac{\frac{-2}{\sqrt{5}}-\frac{3}{\sqrt{5}}}{\sqrt{5}}, \frac{\frac{-1}{\sqrt{5}}+\frac{3}{\sqrt{5}}}{\sqrt{5}}\right) = \left(\frac{-6}{5}, \frac{7}{5}\right)$ and the

directrixs in $(x'y')$ system according to $X = x' - \frac{3}{\sqrt{5}}$ are $x' - \frac{3}{\sqrt{5}} = \pm \frac{2}{\sqrt{5}}$ ie

$x' - \sqrt{5} = 0$ and $x' - \frac{1}{\sqrt{5}} = 0$

Now for the directrixes in (x, y) system, substituting $x = x'\cos\theta + y\sin\theta = \frac{2x+y}{\sqrt{5}}$,

the original directrixs are $\frac{2x+y}{\sqrt{5}} - \sqrt{5} = 0$

and $\frac{2x+y}{\sqrt{5}} - \frac{1}{\sqrt{5}} = 0$ ie $2x+y-5=0$ and

$2x+y-1=0$

Thus the given second degree equation represents rectangular hyperbola

The eccentricity is $e = \sqrt{2}$

The given Co-ordinate of the focci $(2,3)$ and $\left(-\frac{6}{5}, \frac{7}{5}\right)$.

The equation of the directrixs are $2x+y-5=0$ and $2x+y-1=0$

and the length of both axes are $4\sqrt{\frac{2}{5}}$ unit.

[OR] 20]

11) Here $a = b$

$\theta = \pi/4$

$\therefore$ On rotating axes by $\theta = \pi/4$

$x = \frac{x' - y'}{\sqrt{2}}$ $y = \frac{x' + y'}{\sqrt{2}}$

$\therefore$ the equation of the curve are

$\left(\frac{x' - y'}{\sqrt{2}}\right)^2 + \left(\frac{x' + y'}{\sqrt{2}}\right)^2 - 4\left(\frac{x' - y'}{\sqrt{2}}\right) - 6\left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right)$

$-2 = 0$

$\therefore x'^2 + y'^2 - 5\sqrt{2}x' - \sqrt{2}y' - 2 = 0$

$\therefore x'^2 - 5\sqrt{2}x' + \frac{25}{2} + y'^2 - \sqrt{2}y' + \frac{1}{2} = 15$

$\therefore \left(x' - \frac{5}{\sqrt{2}}\right)^2 + \left(y' - \frac{1}{\sqrt{2}}\right)^2 = 15$

Now, on shifting the origin to $\left(\frac{5}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$x' - \frac{5}{\sqrt{2}} = X$ and $y' - \frac{1}{\sqrt{2}} = Y$

$\therefore$ the equation of the curve is

$X^2 + Y^2 = (\sqrt{15})^2$, which represent a circle

21

2) Here $a \neq b$

$\therefore$ On rotating the axes by an angle θ

$\tan 2\theta = \frac{2h}{a-b} = \frac{0}{1+1} = 0$

$\therefore \frac{2\tan\theta}{1-\tan^2\theta} = 0$

$\therefore \tan\theta = 0$

$\therefore \theta = 0$

$\therefore$ Hence there is no need to dotates the axes

Here the equation is,

$x^2 + 4x + 4 - y^2 + 2y - 1 = 0$

$\therefore (x+2)^2 - (y-1)^2 = 0$

Now, on shifting the origin to $(-2, 1)$

$x+2 = x'$ and $y-1 = y'$

$\therefore$ the equation of the curve is

$x'^2 - y'^2 = 0$ ie $(x'+y')(x'-y') = 0$, which represent a pair of lines.

Here we get the lines $x'+y' = 0$ & $x'-y' = 0$

Now, the original lines in (x, y) system are

$x+2+y-1 = 0$ and $x+2-y+1 = 0$

ie $x+y+1 = 0$ and $x-y+3 = 0$

Thus, the given second degree equ. represent a pair of lines, whose equation are $x+y+1=0$ & $x-y+3=0$

B(1) Text page 154 Theorem -5 22]

2) Suppose $\bar{x} = (x_1, x_2, x_3)$ $\bar{y} = (y_1, y_2, y_3)$ and $\bar{z} = (z_1, z_2, z_3) \in R^3$

Here, we want to show that $\bar{x}+\bar{y}, y+\bar{z}, \bar{z}+\bar{x}$ are not coplanar vectors..

Also. $\bar{x} \neq \theta, \bar{y} \neq \theta, \bar{z} \neq \theta$, and x, y and $\bar{z}$ are non-coplanar is given

$\therefore \bar{x} \cdot (\bar{y} \times \bar{z}) = [x\ y\ \bar{z}] = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} \neq 0$

Now, $(\bar{x}+\bar{y}) \cdot [(y+\bar{z}) \times (\bar{z}+\bar{x})]$

$= (\bar{x}+\bar{y}) [y \times \bar{z} + y \times \bar{x} + \bar{z} \times \bar{z} + \bar{z} \times \bar{x}]$

$= (\bar{x}+\bar{y}) [y \times \bar{z} + y \times \bar{x} + \theta + \bar{z} \times \bar{x}]$

$= (\bar{x}+\bar{y}) [y \times \bar{z} + y \times \bar{x} + \bar{z} \times \bar{x}]$

$= x \cdot (\bar{y} \times \bar{z}) + x \cdot (y \times \bar{x}) + x \cdot (\bar{z} \times \bar{x}) + y (\bar{y} \times \bar{z})$
$+ y (\bar{y} \times \bar{x}) + y (\bar{z} \times \bar{x})]$

$= [\bar{x}\ y\ \bar{z}] + [\bar{x}\ y\ \bar{x}] + [\bar{x}\ \bar{z}\ \bar{x}] + [y\ \bar{y}\ \bar{z}]$
$+ [y\ y\ \bar{x}] + [y\ \bar{z}\ \bar{x}]$

$= [\bar{x}\ y\ \bar{z}] + [\bar{x}\ y\ \bar{z}]$

$= 2[\bar{x}\ y\ \bar{z}]$

$\neq 0$ ($\because$ data

$\therefore$ the vector $x+y, y+\bar{z}$ and $\bar{z}+\bar{x}$ are non coplan

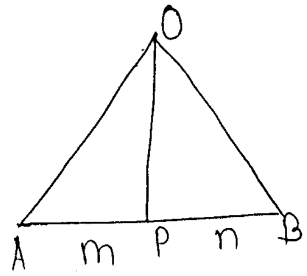
23]

C (1) Text page 175

2) Here the direction of ~~rays~~ vectors $\overrightarrow{AP}$ and $\overrightarrow{PB}$ are same and $\frac{AP}{PB} = \frac{m}{n}$. Hence $n\,\overrightarrow{AP} = m\,\overrightarrow{PB}$

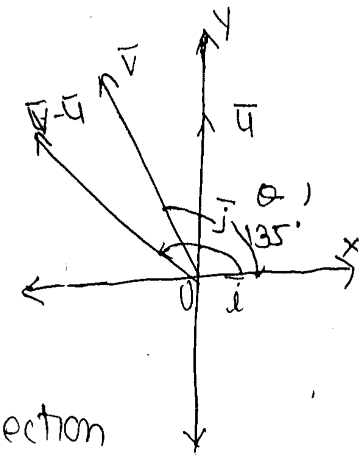
$\therefore n(\overrightarrow{OP} - \overrightarrow{OA}) = m(\overrightarrow{OB} - \overrightarrow{OP})$

$\therefore (m+n)\,\overrightarrow{OP} = n(\overrightarrow{OA}) + m(\overrightarrow{OB})$



B (1) Here, the velocity of the boat is $\bar{a} = 0\cdot\bar{i} + 6\sqrt{2}\,\bar{j}$

$= \frac{12}{\sqrt{2}}\,\bar{j}$



Suppose the true velocity of the wind is $\bar{v}$. The wind blows from the south-East. i.e. It seems to go in the direction North-West and the velocity of the wind relative of the boat is

$\bar{v} - \bar{u} = 5\cos 135°\,\bar{i} + 5\sin 135°\,\bar{j}$

$\therefore \bar{v} - \bar{u} = -\frac{5}{\sqrt{2}}\,\bar{i} + \frac{5}{\sqrt{2}}\,\bar{j}$

Now the true velocity of wind $\bar{v} = (\bar{v} - \bar{u}) + \bar{u}$

$\therefore \bar{v} = \frac{-5}{\sqrt{2}}\,\bar{i} + \frac{17}{\sqrt{2}}\,\bar{j}$

Now $|\bar{v}| = \sqrt{\frac{25}{2} + \frac{289}{2}} = \sqrt{157}$

24

~~Q~~ and if $\bar{v}$ makes an angle θ with $\vec{OX}$, then

$$\cos\theta = \frac{\bar{v}\cdot\hat{i}}{|\bar{v}|\,|\hat{i}|} = \frac{-5}{\sqrt{2}\times\sqrt{157}\times 1} = \frac{-5}{\sqrt{314}}$$

$$\therefore \theta = \cos^{-1}\left(\frac{-5}{\sqrt{314}}\right) = \pi - \cos^{-1}\left(\frac{5}{\sqrt{314}}\right)$$

$\therefore$ the magnitude of the true velocity of the wind is $\sqrt{157}$ ~~units~~ units and its direction is at angle $\pi - \cos^{-1}\left(\frac{5}{\sqrt{314}}\right)$ with the East towards the North.

2) Here $(2a, a, -4)\cdot(a, -2, 1) = 0$

$$\therefore 2a^2 - 2a - 4 = 0$$

$$\therefore a^2 - a - 2 = 0$$

$$\therefore (a-2)(a+1) = 0$$

$$\therefore a-2 = 0 \text{ or } a+1 = 0$$

$$\therefore a = 2 \text{ or } a = -1$$

5(A)(1) Text Page 191

(2) Text Page 2.4

OR

Text Page 203

B (1) Test Page 214 (3)

B (2) 25

Here $l+m+n=0$ ... (1)

and $l^2-m^2+n^2=0$ ... (2)

Now from the equation (1), we get $m=-(l+n)$.

Substituting it in the equation (2),

$l^2-[-(l+n)]^2+n^2=0$

$\therefore l^2-(l+n)^2+n^2=0$

$\therefore l^2-(l^2+2ln+n^2)+n^2=0$

$\therefore l^2-l^2-2ln-n^2+n^2=0$

$\therefore -2ln=0$

$\therefore ln=0$

$\therefore l=0$, or $n=0$

Now:-

1) If $l=0$, then from the equation (1) $m=-n$ and so we get the direction ratios of the first line $0, -n$ and n.

2) If $n=0$ then from the equation (2) $m=-l$ and so we get the direction ratios of the second line $l, -l$ and 0.

Now, for angle θ bet^n two lines 26)

$$\cos\theta = \frac{(0,-n,n)\cdot(l,-l,0)}{\sqrt{2n^2}\ \sqrt{2l^2}}$$

$$= \frac{nl}{2nl}$$

$$= \frac{1}{2}$$

$$\therefore \theta = \pi/3$$

OR

Here vector eq^n from given cartesian eq^n of line is $\bar{r} = (3,-15,9) + k(2,-7,5)$ and $\bar{r} = (-1,1,9) + k(2,+1,-3)$

comparing them with vector eq^n $\bar{r} = \bar{a} + k\bar{l}$ and $\bar{r} = \bar{b} + k\bar{m}$, we get

$\bar{a} = (3,-15,9)$ $\bar{b} = (-1,1,9)$ $\bar{l} = (2,-7,5)$ and $\bar{m} = (2,1,-3)$

$$\therefore \bar{l}\times\bar{m} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$$

$$= \bar{i}(21-5) - \bar{j}(-6-10) + \bar{k}(2+14)$$

$$= 16\bar{i} + 16\bar{j} + 16\bar{k}$$

$$= (16,16,16)$$

$$\therefore |\bar{l}\times\bar{m}| = \sqrt{(16)^2+(16)^2+(16)^2}$$

$$= 16\sqrt{3}$$

Now, $\bar{u} = \dfrac{\bar{l}\times\bar{m}}{|\bar{l}\times\bar{m}|}$

$$= \frac{(16,16,16)}{16\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}}(1,1,1)$$

and $\bar{a} - \bar{b} = (3, -15, 9) - (-1, 1, 9)$ 27

$= (4, -16, 0)$

$\therefore$ ⊥r dist. betn two lines is

$|(\bar{a} - \bar{b}) \cdot \bar{u}| = |(4, -16, 0) \cdot \frac{1}{\sqrt{3}}(1, 1, 1)|$

$= \frac{1}{\sqrt{3}} |4 - 16 + 0|$

$= \frac{1}{\sqrt{3}} |-12| = \frac{12}{\sqrt{3}} = 4\sqrt{3}$

$\therefore$ Thus, the shortest dist. betn two lines is $4\sqrt{3}$ units

C

(1) Here supp. $V(4, 5, 1)$ $A(0, -1, -1)$ $B(3, 9, 4)$ & $C(-4, 4, 4)$

$\therefore \overrightarrow{VA} = (-4, -6, -2)$, $\overrightarrow{VB} = (-1, 4, 3)$, $\overrightarrow{VC} = (-8, -1, 3)$

Now, $\overrightarrow{VA} \cdot (\overrightarrow{VB} \times \overrightarrow{VC}) = [\overrightarrow{VA}\ \overrightarrow{VB}\ \overrightarrow{VC}]$

$= \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$

$= -4(12+3) + 6(-3+24) - 2(1+32)$

$= -60 + 126 - 66$

$= 0$

$\therefore \overrightarrow{VA}, \overrightarrow{VB}$ & $\overrightarrow{VC}$ are collinear

$\therefore$ the pts. V, A, B & C are coplanar

$\therefore$ they cannot be vertices of any tetrahedron.

(2) Let required eqn of a sphere

$S = x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$

$(0, 0, 0) \in S \quad \therefore d = 0$

$(a, 0, 0) \in S \quad \therefore a^2 + 2ua = 0 \Rightarrow u = -a/2 \ (a \neq 0)$

28

$(0,b,0) \in S \quad \therefore b^2 + 2bv = 0 \Rightarrow v = -\frac{b}{2} \; (b \neq 0)$

$(0,0,c) \in S \quad \therefore c^2 + 2wc = 0 \Rightarrow w = -\frac{c}{2}$

$\therefore$ The eq^n of sphere will be

$x^2 + y^2 + z^2 - ax - by - cz = 0$

centre $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$

& $r = \frac{1}{2}\sqrt{a^2+b^2+c^2}$.

(D)

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$

Here eq^n of lines are $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$

and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$

$\therefore \bar{a} = (4,-3,-1)$, $\bar{b} = (1,-1,-10)$, $\bar{l} = (1,-4,7)$

and $\bar{m} = (2,-3,8)$

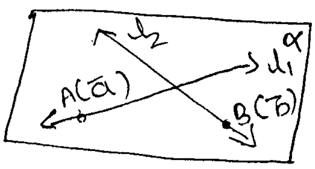
Now, $\bar{l} \times \bar{m} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -4 & 7 \\ 2 & -3 & 8 \end{vmatrix}$

$= \bar{i}(-32+21) - \bar{j}(8-14) + \bar{k}(-3+8)$

$= \bar{i}(-11) - \bar{j}(-6) + \bar{k}(5)$

$= -11\bar{i} + 6\bar{j} + 5\bar{k}$

$= (-11, 6, 5)$



and $\bar{b} - \bar{a} = (1,-1,-10) - (4,-3,-1)$

$= (-3, 2, -9)$

29

$\therefore (\bar{b}-\bar{a})\cdot(\bar{l}\times\bar{m}) = (-3,2,-9)\cdot(-11,6,5)$

$= 33+12-45 = 0$

$\therefore$ both lines are intersecting lines

Now, the eqn of the plane passing through these lines, acc. to

$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$ is,

$\begin{vmatrix} x-4 & y+3 & z+1 \\ 1 & -4 & 7 \\ 2 & -3 & 8 \end{vmatrix} = 0$

$\therefore (x-4)(-32+21) - (y+3)(8-14) + (z+1)(-3+8) = 0$

$\therefore (x-4)(-11) - (y+3)(-6) + (z+1)5 = 0$

$\therefore -11x + 44 + 6y + 18 + 5z + 5 = 0$

$\therefore -11x + 6y + 5z + 67 = 0$

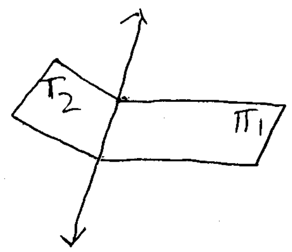
OR

Here the planes are $x+y+2z=4$ and $2x-y+z=-1$

ie $(x,y,z)\cdot(1,1,2)=4$ and $(x,y,z)\cdot(2,-1,1)=-1$

Comparing these eqn with gen. eqn $\bar{r}\cdot\bar{n}=d$ of the plane.

For first plane $\bar{n_1} = (1,1,2)$ and for the second plane, $\bar{n_2} = (2,-1,1)$



Now, $\bar{n} = \bar{n_1}\times\bar{n_2}$

$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$.

$= \bar{i}(1+2) - \bar{j}(1-4) + \bar{k}(-1,-2)$

$= 3\bar{i} + 3\bar{j} - 3\bar{k}$

$= (3,3,-3)$

Now, to obtain common pt. of intersection for both the planes taking $z=0$ in their eq^n we get $x+y=4$ & $2x-y=1$

On solving eq^n we get $x=1$ & $y=3$

Thus one common pt. of intersection is $\bar{a} = (1,3,0)$

Also, eq^n of intersecting line of planes passing through $\bar{a} = (1,3,0)$ & having dir^n $\bar{n} = (3,3,-3)$

acc. to $\bar{r} = \bar{a} + K_1 \cdot \bar{n}$ $(K_1 \in R)$ is

$$\bar{r} = (1,3,0) + K_1(3,3,-3) \quad (K \in R)$$
$$= (1,3,0) + 3K_1(1,1,-1); \quad (K \in R)$$
$$= (1,3,0) + K(1,1,-1); \quad (K \in R)$$
$$(\because \text{taking } 3K_1 = K)$$

Thus eq^n of intersecting line of planes is

$\bar{r} = (1,3,0) + K(1,1,-1)$; $K \in R$.