Q. Paperset No. 4

Mathematics I (050)(E) 1

Maximum Marks 75.

Time : 3 Hrs.

Q1.(A) ① Using co-ordinate geometry in R^2 obtain the co-ordinates of incentre of tringle. (3)

(2) If A is (2, 3) and B is (0, 7) in what ratio does the X-axis divides $\overline{AB}$ from B. (1)

(B) Answer any Two :- (4)

① If A, B, C, P are distinct and non-collinear points of the plane then prove that area of ΔPAB + area of ΔPBC + area of $\Delta PCA \geq$ area of ΔABC.

② Find point C on the $\overleftrightarrow{AB}$ such that AB = 3AC where A (0, 1), B (2, 9).

③ If (3, 2), (4, 5) and (2, 3) are three of the four vertices of a parallelogram, find the co-ordinates of fourth vertex. (4)

(C) Attempt any two :-

① If A (3, 2), B (5, 6) and $P(x, y) \in \overline{AB}$ then prove that $17 \leq 3x + 4y \leq 39$.

② Find the equation of line which passes through (3, 4) and which makes an angle of $\frac{\pi}{4}$ with the line $3x + 4y - 2 = 0$.

③ Prove that the points (3, 4) and (−2, 1) are on opposite side of the line $3x - y + 6 = 0$ in R^2 (3)

(D) Using slopes of two intersecting lines obtain the formula for measure of angle between them. If one out of two intersecting line is vertical then what is the form of measure of angle between them? (3)

Q2.(A) ① Prove that if general quadratic equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of lines then this pair is parallel to the pair of lines represented by $ax^2 + 2hxy + by^2 = 0$ (where $a^2 + h^2 + b^2 \neq 0$)

② Find the angle between the lines represented by $x^2 - 2xy\sec\alpha + y^2 = 0$, $(0 < \alpha < \frac{\pi}{2})$. (1)

2.

(B) ① Find the condition for the line $y = mx + c$ be tangent to the circle $x^2+y^2=r^2$ and the point of contact. (2)

② The sides of a triangle are along the lines $2x-3y+5=0$ and $3x+2y+7=0$ and $x=2$ find the point of concurrence of all the three altitudes of such triangle. ①

③ If $px^2+3y^2+(q-3)xy+2px+3qy-3=0$ represents a circle then find centre and radius. ①

(C) ① Find the equation of the circle passing through the points $(5,-8)$, $(-2,9)$ and $(2,1)$. (3)

OR

① Find the equation of the tangents to the circle $x^2+y^2=17$ from the point $(5,3)$.

② Show that for $\lambda \in R-\{a\}$ line $\frac{x}{a-\lambda}+\frac{y}{b}=1$ passes through a fixed point. ①

(D) Find the area of the parallelogram whose sides are along the lines $y=mx+a$, $y=mx+b$, $y=nx+c$, $y=nx+d$. (3)

OR

(D) Show that lines $ax+by+c=0$, $bx+cy+a=0$, $cx+ay+b=0$ are concurrent and also find point of concurrence. ($b^2\neq ac$, $c^2\neq ab$, $a^2\neq bc$)

Q-3 (A) ① Obtain standard equation of parabola. (2)

② If the focus of the parabola $y^2=4ax$ divides a focal chord in the ratio 1:2 then find the equation of the line containing the focal chord. (2)

OR

② Show that the line $3y=6x+2$ touches the parabola $3y^2=16x$. Find the point of contact.

(B) ① Obtain the equation of the tangent at the point (x_1, y_1) of the ellipse and hence obtain the equation of the tangent at θ-point of the ellipse. (2)

② If the difference of the eccentric angle of P and Q points on the ellipse is $\pi/2$ and O is the origin then prove that the area of ΔPOQ is $\frac{1}{2}ab$. (2)

OR

3.

OR

② The tangent at the point P intersects a directrix at F. Prove that PF forms a right angle at the corresponding focus.

(C) ① Define rectangular hyperbola. Obtain its standard equation and eccentricity. (2)

② Show that the angle between two asymptotes of the hyperbola $x^2-2y^2=1$ is $\tan^{-1}2\sqrt{2}$. (2)

(D) ① If $S(4,0)$ and $e=3/2$, find the equation of a hyperbola. ①

② Find the set of all points P outside a circle, such that the tangent drawn to a circle from P are perpendicular to each-other. (2)

Q4 (A) Which curve is represented by the equation $3x^2+8xy-3y^2-20x+10y-15=0$. Find the co-ordinates of focii, equation of directrices and eccentricity. (4)

OR

Identify the following curves by obtaining their standard form of equation.

(i) $x^2+y^2-4x-6y-2=0$

(ii) $x^2-y^2+4x+2y+3=0$.

(B) ① Obtain necessary and sufficient condition for two vectors $\bar{x}, \bar{y} \in R^2$ to be collinear. $(\bar{x} \neq 0, \bar{y} \neq 0)$. (2)

② If $\bar{x}, \bar{y}, \bar{z}$ are linearly independent then prove that $\bar{x}+\bar{y}$, $\bar{y}+\bar{z}$, $\bar{z}+\bar{x}$ are also linearly independent (2)

(C) ① Obtain formula for the volume of a prism. (using vectors) (2)

② If A - P - B and if $AP:PB = m:n$ then, for any point O - in space prove that $n\vec{OA}+m\vec{OB}=(m+n)\vec{OP}$. (2)

(D) ① A boat speeds in the north at $6\sqrt{2}$ kms. A man on the boat feels that the wind is blowing from the south-east at 5 kms Find the true velocity of the wind. (2)

② Find the value of a, if $(2a\bar{i}+a\bar{j}+4\bar{k}) \perp (a\bar{i}-2\bar{j}-\bar{k})$ (1)

4.

Q.5 (A) ① In usual notations obtain the distance between given point and given line not containing that point in R^3. (2)

② Obtain equation of a plane passing through two intersecting lines in R^3. (2)

OR

② Obtain equation of a plane passing through two parallel lines in R^3.

(B) ① Find vector and cartesian equation of a sphere having centre $C(\bar{c})$ and radius r. (1)

② If the direction cosines l, m, n of two lines satisfy $l+m+n=0$ and $l^2+n^2=m^2$, show that the angle between the two lines is $\pi/3$. (3)

OR

② obtain shortest distance between the lines $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$ and $\frac{x+1}{2} = \frac{y-1}{1} = \frac{9-z}{3}$.

(C) ① Show that $(4, 5, 1)$, $(0, -1, -1)$, $(3, 9, 4)$, $(-4, 4, 4)$ can not be vertices of any tetrahedron. (2)

② obtain the equation, the centre and radius of the sphere through $(0, 0, 0)$, $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$. (2)

(D) obtain the equation of the plane passing through $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$. (3)

OR

Obtain the intersection of the plane $x+y+2z=4$ and $2x-y+z+1=0$.

— x — x —

Solution of paper set No. 4 (1)

Mathematics I (050)(E)

A.1 (A) (1) Theory (Text) page No. 19.

(2) Here A is (2, 3) and B is (0, 7)

Suppose the point $P(x, 0)$ of the X-axis divides $\overline{AB}$ from B in the ratio $m : n$ where $m+n \neq 0$

$\therefore$ according to the y- co-ordinate,

$$y = \frac{my_2 + ny_1}{m+n} \text{ of } P,$$

$$\therefore 0 = \frac{3m + 7n}{m+n}$$

$$\therefore 3m = -7n$$

$$\therefore m : n = -7 : 3$$

$\therefore$ The X-axis divides $\overline{AB}$ from B at point $P(x, 0)$ in the ratio $-7 : 3$.

(B) (1) Suppose co-ordinates of point P are (0, 0) and corresponding to that co-ordinates of A, B & C are (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively.

For ΔABC, $D = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$.

If determinants corresponding to ΔPAB, ΔPBC and ΔPCA are D_1, D_2 and D_3 respectively, then

$$D_1 + D_2 + D_3$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 1 \\ x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \end{vmatrix}$$

$$= (x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3).$$

$$= x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_1 - x_1y_3$$

$$= x_1(y_2 - y_3) - y_1(x_2 - x_3) + 1(x_2y_3 - x_3y_2).$$

$$= \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= D.$$

Thus, $D = D_1 + D_2 + D_3$.

$$\therefore |D| = |D_1 + D_2 + D_3|$$

$$\therefore |D| \leq |D_1| + |D_2| + |D_3|$$

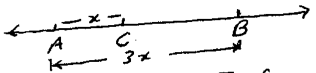
$$\therefore \tfrac{1}{2}|D| \leq \tfrac{1}{2}|D_1| + \tfrac{1}{2}|D_2| + \tfrac{1}{2}|D_3|,$$

$\therefore$ The area of $\Delta ABC \leq$ area of ΔPAB + area of ΔPBC + area of ΔPCA,

2.

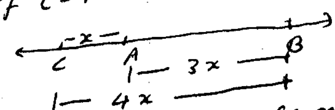
(2) Here $A(0, 1)$ and $B(2, 9)$ and suppose $C(x, y)$.
Now A, B and C are collinear and $AB = 3AC$
$\therefore$ There are two possibilities.

Case(i) If $A-C-B$ then,



$\therefore$ C divides $\overline{AB}$ from A in the ratio $\lambda = \frac{AC}{CB} = \frac{x}{2x} = \frac{1}{2}$.
Using division pt. co-ordinates
$x = \frac{mx_2 + nx_1}{m+n}$ & $y = \frac{my_2 + ny_1}{m+n}$
Co-ordinates of C are
$\left(\frac{1(2)+2(0)}{1+2}, \frac{1(9)+2(1)}{1+2}\right)$
$= \left(\frac{2}{3}, \frac{11}{3}\right)$,

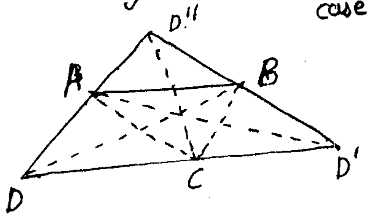
Case (ii)
If $C-A-B$ then,



$\therefore$ C divides $\overline{AB}$ from A in the ratio $\lambda = -\frac{AC}{CB} = -\frac{x}{4x}$
$\therefore \lambda = -\frac{1}{4}$
$\therefore$ Co-ordinates of C are
$\left(\frac{-1(2)+4(0)}{-1+4}, \frac{-1(9)+4(1)}{-1+4}\right)$
$= \left(-\frac{2}{3}, -\frac{5}{3}\right)$.

Thus the co-ordinates of C are $\left(\frac{2}{3}, \frac{11}{3}\right)$ or $\left(-\frac{2}{3}, -\frac{5}{3}\right)$.

(3) Suppose A is $(3, 2)$, B is $(4, 5)$ and C is $(2, 3)$.
We can get the fourth vertex of the parallelogram by three ways.



case ① $\square ABCD$ is a parallelogram and if the co-ordinates of D are (x, y) then the mid-points of the diagonals $\overline{AC}$ and $\overline{BC}$ are same. $\therefore$ $\frac{x+4}{2} = \frac{5}{2}$ and $\frac{y+5}{2} = \frac{5}{2}$
$\therefore$ $x = 1$ and $y = 0$
$\therefore$ $D(1, 0)$.

case ② $\square ABD'C$ is a parallelogram.
$\therefore$ mid pt. diagonal $\overline{AD'}$ = mid pt. of diagonal $\overline{BC}$.
If Co-ordinates of D' are (x', y')
$\frac{x'+3}{2} = \frac{6}{2}$ and $\frac{y'+2}{2} = \frac{8}{2}$
$\therefore$ $x' = 3$ and $y' = 6$
$\therefore$ $D'(3, 6)$.

case ③ $\square ACBD''$ is parallelogram and if co-ordinates of D'' are (x'', y'') then the mid pt. of diagonal $\overline{CD''}$ = mid pt. of diagonal $\overline{AB}$
$\therefore$ $\frac{x''+2}{2} = \frac{7}{2}$ and $\frac{y''+3}{2} = \frac{7}{2}$
$\therefore$ $x'' = 5$ and $y'' = 4$.
$\therefore$ we get $D''(5, 4)$.

$\therefore$ Thus, the fourth vertex of the given parallelogram is $(1, 0)$ or $(3, 6)$ or $(5, 4)$.

3.

(C) (1) For $A(3, 2)$, $B(5, 6)$

Parametric equation of $\overleftrightarrow{AB}$:-

$x = tx_2 + (1-t)x_1$, $y = ty_2 + (1-t)y_1$

$= 5t + (1-t)3$, $= 6t + (1-t)2$, $t \in R$.

$\therefore x = 2t + 3$, $\therefore y = 4t + 2$

$\therefore \quad 3x + 4y = 6t + 9 + 16t + 8$

$\therefore 3x + 4y = 22t + 17$

But $P(x, y) \in \overline{AB}$

$\therefore \quad 0 \le t \le 1$

$\therefore 0 \le 22t \le 22.$

$\therefore 17 \le 22t + 17 \le 39$

$\therefore \quad 17 \le 3x + 4y \le 39.$

(2) Here the slope of the line $3x + 4y - 2 = 0$ is $m_1 = -\frac{3}{4}$

Suppose, the slope of the required line is m_2.

Also, the measure of the angle between these two lines is $\alpha = 45^\circ$

Now by $\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$$\tan 45^\circ = \left|\frac{-3/4 - m_2}{1 + (-3/4)m_2}\right|$$

$$\therefore \quad 1 = \left|\frac{-3 - 4m_2}{4 - 3m_2}\right|$$

$\therefore \quad -3 - 4m_2 = 4 - 3m_2$ or $-3 - 4m_2 = -4 + 3m_2$

$\therefore \quad m_2 = -7 \quad$ or $\quad m_2 = \frac{1}{7}$

$\therefore$ two lines are possible.

Now by slope point equation of a line

$y - y_1 = m(x - x_1)$

$y - 4 = -7(x - 3)$ or $y - 4 = \frac{1}{7}(x - 3)$

$\therefore 7x + y - 25 = 0$ $\quad \therefore x - 7y + 25 = 0.$

Thus, the required lines are $x - 7y + 25 = 0$ and $7x + y - 25 = 0$.

(3) Suppose A is $(3, 4)$ & B is $(-2, 1)$

Suppose $P(x, y) \in l : 3x - y + 6 = 0$ divides $\overline{AB}$ from A in the ratio λ. $(\lambda \ne -1, 0)$

By division point co-ordinates of a division point of line segment co-ordinates of point P are $\left(\frac{\lambda x_2 + x_1}{\lambda + 1}, \frac{\lambda y_2 + y_1}{\lambda + 1}\right) = \left(\frac{-2\lambda + 3}{\lambda + 1}, \frac{\lambda + 4}{\lambda + 1}\right) \in l$.

$\therefore \quad 3\left(\frac{-2\lambda + 3}{\lambda + 1}\right) - \left(\frac{\lambda + 4}{\lambda + 1}\right) + 6 = 0$

$\therefore \quad -6\lambda + 9 - \lambda - 4 + 6\lambda + 6 = 0$

$\therefore \quad \lambda = 11 > 0$

$\therefore$ we get $A - P - B$. $\therefore$ The points $A(3, 4)$ and $B(-2, 1)$ are on the opposite side of the line $3x - y + 6 = 0$.

5

(C) (1) Equation of a circle whose diametrically opposite (end) points are $(-2, 9)$ and $(2, 1)$ is

$(x+2)(x-2)+(y-9)(y-1)=0$

$\therefore\ x^2+y^2-10y+5=0. \quad \leftarrow (1)$

The equation of the line passing through points $(-2, 9)$ and $(2, 1)$ is

$$y-9=\frac{1-9}{2+2}(x+2).$$

$\therefore\ 2x+y-5=0 \quad \leftarrow (2)$

The general equation of the circle passing through the point of intersections of (1) and (2) is

$x^2+y^2-10y+5+\lambda(2x+y-5)=0 \quad \leftarrow (3)$

If this circle is passes through $(5,-8)$

$(25+64+80+5)+\lambda(10-8-5)=0$

$\therefore\ 174-3\lambda=0$

$\therefore\ \lambda=+58.$

Now substituting this value of λ in (3)

Equation of desired circle is

$(x^2+y^2-10y+5)\cancel{+}58(2x+y-5)=0$

$\therefore\ x^2+y^2+116x+48y-285=0$

$\boxed{OR}$

Here $P(5, 3)\notin S: x^2+y^2=17 \quad (\because 25+9=34\neq 17).$

From equation of circle $x^2+y^2=r^2$ given circle have centre $O(0,0)$ and radius $r=\sqrt{17}$

If line $y=mx\pm r\sqrt{1+m^2}$ is tangent to circle passes through $(5, 3)$ then,

$3=5m\pm\sqrt{17}\sqrt{1+m^2}.$

$\therefore\ (3-5m)^2=17(1+m^2)$

$\therefore\ 4m^2-15m-4=0$

$\therefore\ (4m+1)(m-4)=0$

$\therefore\ m=-\frac{1}{4}$ or $m=4$

Now by a slope point equation of a line equation of required tangents to circle are

$y-y_1=m(x-x_1)$	$y-3=4(x-5)$
$\therefore\ y-3=-\frac{1}{4}(x-5)$	$\therefore\ 4x-y-17=0$
$\therefore\ x+4y-17=0$	

Thus the equations of the tangents to the given circle from the given point are $4x-y-17=0$ and $x+4y-17=0$.

(4)

A.1.D. Theory (Text) page No. 36.

A.2.A (1) Theory (Text) page No. 63.

(2) Comparing the equation $x^2 - 2xy\sec\alpha + y^2 = 0$ with the homogeneous quadratic equation $ax^2 + 2hxy + by^2 = 0$, $a = 1$, $h = -\sec\alpha$, $b = 1$.

Here $h^2 - ab = \sec^2\alpha - 1 = \tan^2\alpha > 0$ $(\because 0 < \alpha < \pi/2)$.

$\therefore$ Given equation represents two distinct lines passes through $O(0,0)$.

If the measure of angle between the lines is θ then according to,

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{|a+b|} = \frac{2\sqrt{\sec^2\alpha - 1}}{2} = \tan\alpha. \ (\because 0 < \alpha < \pi/2)$$

$\therefore \theta = \alpha$.

Thus, the measure of the required angle is α unit.

(B) (1) Text page. 76.

(2) Suppose lines $l_1: 2x - 3y + 5 = 0$ —(1), $l_2: 3x + 2y + 7 = 0$ —(2)

Slope of line l_1, $m_1 = \frac{2}{3}$

" " " l_2, $m_2 = -3/2$

$\therefore m_1 \cdot m_2 = -1$.

$\therefore l_1 \perp l_2$.

Solving equations (1) and (2) we get orthocentre H of the triangle.

$$H(x,y) = \left(\frac{-21-10}{4+9}, \frac{15-14}{4+9}\right)$$

$$\therefore H(x,y) = \left(-\frac{31}{13}, \frac{1}{13}\right).$$

(3) Since given equation $px^2 + 3y^2 + (q-3)xy + 2px + 3qy - 3 = 0$ represents a circle then

Co-efficient of $xy = 0$

i.e. $(q-3) = 0$

$\therefore q = 3$

& Co-efficient of x^2 = Co-efficient of y^2

$\therefore p = 3$.

$\therefore$ Equation of the circle is,

$3x^2 + 3y^2 + 6x + 9y - 3 = 0$

$\therefore x^2 + y^2 + 2x + 3y - 1 = 0$

$\therefore g = 1$, $f = \frac{3}{2}$, $c = -1$

$\therefore$ Centre $C(-g, -f) = (-1, -\frac{3}{2})$

& radius $= r = \sqrt{g^2 + f^2 - c}$

$= \sqrt{1 + 9/4 + 1}$

$\therefore r = \frac{\sqrt{17}}{2}$ units.

6

(2) Here for given line : $\frac{x}{a-\lambda} + \frac{y}{b} = 1$

$\therefore \frac{y}{b} - 1 = -\frac{x}{a-\lambda}$

$\therefore y - b = -\frac{b}{a-\lambda}(x-0).$

Comparing with $y - y_1 = m(x - x_1)$.

$m = -\frac{b}{a-\lambda}$ and fixed point on the given line is $(x_1, y_1) = (0, b)$.

(D) Here solving $y = mx + a$ and $y = nx + c$ also $y = mx + a$ and $y = nx + d$ we get

$A\left(\frac{c-a}{m-n}, \frac{mc-na}{m-n}\right)$ and $B\left(\frac{d-a}{m-n}, \frac{md-na}{m-n}\right)$

Now $AB^2 = \left(\frac{c-a}{m-n} - \frac{d-a}{m-n}\right)^2 + \left(\frac{mc-na}{m-n} - \frac{md-na}{m-n}\right)^2.$

$= \left(\frac{c-d}{m-n}\right)^2 + m^2\left(\frac{c-d}{m-n}\right)^2$

$= \left(\frac{c-d}{m-n}\right)^2(1+m^2).$

$\therefore AB = \left|\frac{c-d}{m-n}\right|\sqrt{1+m^2}.$

Also, the perpendicular distance between $\overline{AB}$ and $\overline{CD}$ is $P_1 = \frac{|a-b|}{\sqrt{1+m^2}}$.

Now the area of the parallelogram

$= AB \cdot P_1$

$= \left|\frac{c-d}{m-n}\right|\sqrt{1+m^2} \cdot \frac{|a-b|}{\sqrt{1+m^2}}.$

$= \left|\frac{(a-b)(c-d)}{m-n}\right|.$

OR

Here the lines are $ax + by + c = 0$ —(1)
$bx + cy + a = 0$ —(2)
and $cx + ay + b = 0$ —(3) and $b^2 \neq ac$, $c^2 \neq ab$

and $a^2 \neq bc$.

Now $a_1b_2 - a_2b_1 = ac - b^2 \neq 0$
$a_2b_3 - a_3b_2 = ab - c^2 \neq 0$
$a_1b_3 - a_3b_1 = a^2 - bc \neq 0$ ⟵ (I)

and $D = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ b & c & a \\ c & a & b \end{vmatrix} = 0$
$(\because a+b+c=0)$

$\therefore$ Given lines are concurrent.

Now $a+b+c=0$ then their point of concurrence is $(1, 1)$.

7

A3. A (1) Theory Text. page No. 87

(2) Let $P(at_1^2, 2at_1)$ and $Q(at_2^2, 2at_2)$ are the end points of focal chord of parabola $y^2 = 4ax$. where $S(a, 0)$ divides $\overline{PQ}$ from P in ratio 2:1.

$\therefore$ according to $y = \frac{\lambda y_2 + y_1}{\lambda + 1}$, y - co-ordinates of S is $0 = \frac{2(-\frac{2a}{t_1}) + 2at_1}{2+1}$. $(\because t_1 t_2 = -1)$.

$\therefore \frac{4a}{t_1} = 2at_1$

$\therefore t_1 = \pm\sqrt{2} \quad \therefore t_2 = \mp\frac{1}{\sqrt{2}}$.

(i) For $t_1 = \sqrt{2}$, $P(at_1^2, 2at_1) = (2a, 2\sqrt{2}a)$

Equation of ~~foc~~ line $\overleftrightarrow{PQ}$ is

$\begin{vmatrix} x & y & 1 \\ a & 0 & 1 \\ 2a & 2\sqrt{2}a & 1 \end{vmatrix} = 0 \quad (\because S(a,0) \in \overleftrightarrow{QP})$.

$\therefore -2\sqrt{2}ax + ay + 2\sqrt{2}a^2 = 0$

$\therefore y = 2\sqrt{2}(x-a)$.

(ii) For $t_1 = -\sqrt{2}$, $P(2a, -2\sqrt{2}a)$.

$\therefore$ second equation of the line containing the focal - chord is $y = -2\sqrt{2}(x-a)$.

Thus, the two equation of the focal chord are $y = \pm 2\sqrt{2}(x-a)$.

$\boxed{\text{OR}}$

Here, line $3y = 6x + 2$

$\therefore y = 2x + \frac{2}{3}$.

Comparing with $y = mx + c$

$m = 2, \quad c = \frac{2}{3}$

parabola $3y^2 = 16x \Rightarrow y^2 = \frac{16}{3}x \Rightarrow a = \frac{4}{3}$.

$c = \frac{2}{3}$ and $\frac{a}{m} = \frac{4/3}{2} = \frac{2}{3}$

$\therefore c = \frac{a}{m}$

$\therefore$ given line touches the given parabola point of contact $(\frac{a}{m^2}, \frac{2a}{m}) = (\frac{1}{3}, \frac{4}{3})$.

(B) (1) Text page No. 103

(2) Here the difference of the eccentric angles (8)
of points on the ellipse is $\frac{\pi}{2}$.
we will take $P(\theta) = (a\cos\theta, b\sin\theta)$
and $Q(\frac{\pi}{2}+\theta) = (a\cos(\frac{\pi}{2}+\theta), b\sin(\pi/2+\theta))$
$= (-a\sin\theta, b\cos\theta)$
Also $O(0,0)$ is the centre.
The vertices of the ΔOPQ are $O(0,0)$
$P(a\cos\theta, b\sin\theta)$, $Q(-a\sin\theta, b\cos\theta)$.

$$\therefore D = \begin{vmatrix} 0 & 0 & 1 \\ a\cos\theta & b\sin\theta & 1 \\ -a\sin\theta & b\cos\theta & 1 \end{vmatrix}$$

$= ab(\cos^2\theta + \sin^2\theta)$

$= ab$.

$\therefore$ The area of $\Delta OPQ = \Delta = \frac{1}{2}|D|$
$= \frac{1}{2}ab$. $(a>0, b>0)$

$\boxed{OR}$

The tangent $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ at the point $P(a\cos\theta, b\sin\theta)$
to the ellipse intersects the directrix $x = \frac{a}{e}$
of the ellipse at the point $F(\frac{a}{e}, K)$.

$$\therefore \frac{a}{e}\cdot\frac{\cos\theta}{a} + \frac{K}{b}\sin\theta = 1.$$

$$K = \frac{b(e-\cos\theta)}{e\sin\theta}.$$

$$\therefore F\left(\frac{a}{e}, \frac{b(e-\cos\theta)}{e\sin\theta}\right) \ \& \ S(ae, 0).$$

Now the slope of $\overleftrightarrow{SF}$ $\times$ slope of $\overleftrightarrow{SP}$

$$= \left\{\frac{\frac{b(e-\cos\theta)}{e\sin\theta} - 0}{\frac{a}{e} - ae}\right\} \times \left\{\frac{b\sin\theta - 0}{a\cos\theta - ae}\right\}$$

$$= \frac{b(e-\cos\theta)}{a(1-e^2)\sin\theta} \times \frac{b\sin\theta}{-a(e-\cos\theta)}$$

$$= -\frac{b^2}{a^2(1-e^2)}$$

$$= -\frac{b^2}{b^2} \quad (\because \text{For } a>b,\ b^2 = a^2(1-e^2))$$

$= -1$. $\therefore \overleftrightarrow{SF} \perp \overleftrightarrow{SP}$

$\therefore \overline{PF}$ subtends a right angle at the focus S.

⑨

C (1) Text page 121.

(2) Here equation of the asymptotes $x^2-2y^2=0$ of the hyperbola $x^2-2y^2=1$.

If the angle between them is θ, then

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{|a+b|}.$$

As $a=1$, $h=0$, $b=-2$.

$$\therefore \tan\theta = \frac{2\sqrt{0-(-2)}}{|1-2|} = 2\sqrt{2}.$$

$$\therefore \theta = \tan^{-1}2\sqrt{2}.$$

(D) ① Here focus $S(4,0)=(ae,0)$, $e=\frac{3}{2}$

$$\therefore ae = 4$$

$$\therefore a\cdot\frac{3}{2}=4$$

$$\therefore a=\frac{8}{3} \quad \therefore a^2=\frac{64}{9}$$

$$b^2=-a^2(1-e^2) = a^2(e^2-1)$$

$$=-\frac{64}{9}\left(1-\frac{9}{4}\right)=\frac{64}{9}\left(\frac{9}{4}-1\right)$$

$$b^2=\frac{80}{9}.$$

$\therefore$ Equation of hyperbola is

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$$\therefore \frac{9x^2}{64}-\frac{9y^2}{80}=1.$$

② Suppose $y=mx\pm a\sqrt{1+m^2}$ are the tangents to the circle $x^2+y^2=a^2$ which are passing through a point $P(x_1,y_1)$ out side the circle.

$$\therefore y_1 = mx_1 \pm a\sqrt{1+m^2}$$

$$\therefore (y_1-mx_1)^2 = a^2(1+m^2).$$

$$\therefore (a^2-x_1^2)m^2+2x_1y_1m+(a^2-y_1^2)=0.$$

If m_1 and m_2 are the roots of this quadratic equation in m, then $m_1\cdot m_2 = \frac{a^2-y_1^2}{a^2-x_1^2}$. —①

Now tangents through P are perpendicular to each other taking $m_1\cdot m_2=-1$ in ①

$$\therefore \frac{a^2-y_1^2}{a^2-x_1^2}=-1 \quad \therefore a^2-y_1^2=-a^2+x_1^2$$

$$\therefore x_1^2+y_1^2=2a^2.$$

Thus, the locus of P is the concentric circle $x^2+y^2=2a^2$ with the radius $\sqrt{2}a$.

(10)

A-4(A) Here, $a \neq b$, $2h = 8$

$\therefore \tan 2\theta = \frac{2h}{a-b} = \frac{8}{6} = \frac{4}{3}$.

$\therefore$ On rotating the axes by an angle θ where $\tan 2\theta = \frac{4}{3}$.

$\therefore \cos 2\theta = \frac{3}{5}$.

$\therefore \cos\theta = \sqrt{\frac{1+3/5}{2}} = \frac{2}{\sqrt{5}}$, $\sin\theta = \frac{1}{\sqrt{5}}$.

Here $x = x'\cos\theta - y'\sin\theta = \frac{2x'-y'}{\sqrt{5}}$

$y = x'\sin\theta + y'\cos\theta = \frac{x'+2y'}{\sqrt{5}}$

$\therefore$ Equation of curve in (x', y') co-ordinates system is.

$$3\left(\frac{2x'-y'}{\sqrt{5}}\right)^2 + 8\left(\frac{2x'-y'}{\sqrt{5}}\right)\left(\frac{x'+2y'}{\sqrt{5}}\right) - 3\left(\frac{x'+2y'}{\sqrt{5}}\right)^2 - 20\left(\frac{2x'-y'}{\sqrt{5}}\right) + 10\left(\frac{x'+2y'}{\sqrt{5}}\right) - 15 = 0.$$

$$\therefore \frac{1}{5}(12+16-3)x'^2 + \frac{1}{5}(3-16-12)y'^2 + \frac{1}{\sqrt{5}}(-40+10)x' + \frac{1}{\sqrt{5}}(20+20)y' - 15 = 0$$

$$\therefore 5x'^2 - 5y'^2 - 6\sqrt{5}x' + 8\sqrt{5}y' - 15 = 0$$

$$\therefore 5\left(x'^2 - \frac{6x'}{\sqrt{5}} + \frac{9}{5}\right) - 5\left(y'^2 - \frac{8y'}{\sqrt{5}} + \frac{16}{5}\right) = 8$$

$$\therefore \left(x' - \frac{3}{\sqrt{5}}\right)^2 - \left(y' - \frac{4}{\sqrt{5}}\right)^2 = \frac{8}{5}.$$

Now on shifting the origin to $\left(\frac{3}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$,

New co-ordinates are (X, Y) then

$x' - \frac{3}{\sqrt{5}} = X$ and $y' - \frac{4}{\sqrt{5}} = Y$

$\therefore$ The equation of the curve

$$X^2 - Y^2 = \frac{8}{5} = \left(\frac{2\sqrt{2}}{\sqrt{5}}\right)^2 = a^2$$

$\therefore$ The given represents the rectangular hyperbola.

$\therefore a = 2\sqrt{\frac{2}{5}}$ & $e = \sqrt{2}$

In (X, Y) system

Focii : $(\pm ae, 0)$

$= \left(\pm \frac{4}{\sqrt{5}}, 0\right)$.

Directrices:-

$X = \pm \frac{a}{e}$

$X = \pm \frac{2}{\sqrt{5}}$.

Length of Transverse and conjugate axes $= 2a = 4\sqrt{\frac{2}{5}}$ units.

In (x', y') System

Focii :- (x', y')

$= \left(X + \frac{3}{\sqrt{5}}, Y + \frac{4}{\sqrt{5}}\right)$

$= \left(\pm\frac{4}{\sqrt{5}} + \frac{3}{\sqrt{5}}, 0 + \frac{4}{\sqrt{5}}\right)$.

$= \left(\frac{7}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$ and $\left(-\frac{1}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$.

Directrices: -

$X = \pm \frac{2}{\sqrt{5}}$

$\therefore x' - \frac{3}{\sqrt{5}} = \pm \frac{2}{\sqrt{5}}$

$\therefore x' = \sqrt{5}$ & $x' = \frac{1}{\sqrt{5}}$

In (x, y) (i.e. original) system.

Focii : (x, y)

$= \left(\frac{2x'-y'}{\sqrt{5}}, \frac{x'+2y'}{\sqrt{5}}\right)$

$= \left(\frac{2\left(\frac{7}{\sqrt{5}}\right) - \frac{4}{\sqrt{5}}}{\sqrt{5}}, \frac{7/\sqrt{5} + 4/\sqrt{5} \times 2}{\sqrt{5}}\right)$

and $\left(\frac{2\left(-\frac{1}{\sqrt{5}}\right) - \frac{4}{\sqrt{5}}}{\sqrt{5}}, \frac{-\frac{1}{\sqrt{5}} + 2\left(\frac{4}{\sqrt{5}}\right)}{\sqrt{5}}\right)$

$= (2, 3)$ and $\left(-\frac{6}{5}, \frac{7}{5}\right)$.

As $x' = x\cos\theta + y\sin\theta = \frac{2x+y}{\sqrt{5}}$

Equation of directrices,

$x' = \sqrt{5}$ and $x' = \frac{1}{\sqrt{5}}$

$2x + y = 5$ and $2x + y = 1$.

$\therefore 2x + y - 5 = 0$ and $2x + y - 1 = 0$.

OR (11)

(i) Here $a = b \Rightarrow \tan 2\theta = \infty \Rightarrow \theta = \pi/4$.

$\therefore$ On rotating the axes by $\theta = \pi/4$,

$x = \frac{x'-y'}{\sqrt{2}}$ and $y = \frac{x'+y'}{\sqrt{2}}$

$\therefore$ The equation of the curve

$\left(\frac{x'-y'}{\sqrt{2}}\right)^2 + \left(\frac{x'+y'}{\sqrt{2}}\right)^2 - 4\left(\frac{x'-y'}{\sqrt{2}}\right) - 6\left(\frac{x'+y'}{\sqrt{2}}\right) - 2 = 0$.

$\therefore x'^2 + y'^2 - 5\sqrt{2}x' - \sqrt{2}y' - 2 = 0$

$\therefore \cancel{x'^2 - 5\sqrt{2}x' + \frac{25}{2} + y'^2 - \sqrt{2}y' + \frac{4}{\sqrt{2}} = 0}$

$\therefore x'^2 - 5\sqrt{2}x' + \frac{25}{2} + y'^2 - \sqrt{2}y' + \frac{1}{2} = 15$.

$\therefore \left(x' - \frac{5}{\sqrt{2}}\right)^2 + \left(y' - \frac{1}{\sqrt{2}}\right)^2 = 15$.

Now, on shifting the origin to $\left(\frac{5}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

$x' - \frac{5}{\sqrt{2}} = X$, $y' - \frac{1}{\sqrt{2}} = Y$.

$\therefore$ The equation of the curve is

$X^2 + Y^2 = (\sqrt{15})^2$

Which represents a circle.

centre $C(0,0)$ & radius $r = \sqrt{15}$

In (x', y') system centre (x', y')
$= \left(X + \frac{5}{\sqrt{2}}, Y + \frac{1}{\sqrt{2}}\right)$
$= \left(\frac{5}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

In (x, y) system centre $= (x, y)$
$= \left(\frac{x'-y'}{\sqrt{2}}, \frac{x'+y'}{\sqrt{2}}\right)$
$= \left(\frac{\frac{5}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{\sqrt{2}}, \frac{\frac{5}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{\sqrt{2}}\right)$
$= (2, 3)$.

(12) Here $a \neq b$ and $2h = 0$

$\tan 2\theta = 0$

$\therefore \theta = 0$

Hence, there is no need to rotate the axes. Here, the equation $x^2 + 4x + 4 - y^2 + 2y - 1 = 0$

$\therefore (x+2)^2 - (y-1)^2 = 0$

Now on shifting the origin $(-2, 1)$.

$x + 2 = x'$ and $y - 1 = y'$.

$\therefore$ The equation of the curve is $x'^2 - y'^2 = 0$.

i.e. $(x'+y')(x'-y') = 0$ which represents a pair of lines.

Here we get the lines $x'+y' = 0$ and $x'-y' = 0$

Now original lines are $x+y+1 = 0$ and $x-y+3 = 0$.

12.

B (1) Text page 154. Thm.5.

(2) Suppose $\bar{x} = (x_1, x_2, x_3)$, $\bar{y} = (y_1, y_2, y_3)$ and $\bar{z} = (z_1, z_2, z_3) \in R^3$

Here, we want to show that $\bar{x}+\bar{y}$, $\bar{y}+\bar{z}$, $\bar{z}+\bar{x}$ are linearly independent i.e. non-coplanes, ~~is given~~

Also $\bar{x}$, $\bar{y}$ & $\bar{z}$ are linearly independent non null vectors then $\bar{x}\cdot(\bar{y}\times\bar{z}) = 0$. —(1)

$\therefore [\bar{x}\ \bar{y}\ \bar{z}] = 0$.

Now $(\bar{x}+\bar{y})\cdot[(\bar{y}+\bar{z})\times(\bar{z}\times\bar{x})]$.

$= (\bar{x}+\bar{y})\cdot[\bar{y}\times\bar{z} + \bar{y}\times\bar{x} + \theta + \bar{z}\times\bar{x}]$.

$= (\bar{x}+\bar{y})\cdot[\bar{y}\times\bar{z} + \bar{y}\times\bar{x} + \bar{z}\times\bar{x}]$

$= (\bar{x}+\bar{y})\cdot[\bar{y}\times\bar{z} + \bar{y}\times\bar{x} + \bar{z}\times\bar{x}] + \bar{y}\cdot(\bar{y}\times\bar{z}) + \bar{y}\cdot(\bar{y}\times\bar{x}) + \bar{y}\cdot(\bar{z}\times\bar{x})$.

$= \bar{x}\cdot(\bar{y}\times\bar{z}) + \bar{x}\cdot(\bar{y}\times\bar{x}) + \bar{x}\cdot(\bar{z}\times\bar{x}) +$

$= [\bar{x}\ \bar{y}\ \bar{z}] + [\bar{x}\ \bar{y}\ \bar{x}] + [\bar{x}\ \bar{z}\ \bar{x}] + [\bar{y}\ \bar{y}\ \bar{z}] + [\bar{y}\ \bar{y}\ \bar{x}] + [\bar{y}\ \bar{z}\ \bar{x}]$.

$= [\bar{x}\ \bar{y}\ \bar{z}] + 0 + 0 + 0 + 0 + [\bar{y}\ \bar{z}\ \bar{x}]$.

$= [\bar{x}\ \bar{y}\ \bar{z}] + [\bar{x}\ \bar{y}\ \bar{z}]$

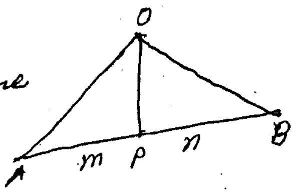
$= 2[\bar{x}\ \bar{y}\ \bar{z}]$.

$\neq 0$. (By 1)

$\therefore$ The vectors $\bar{x}+\bar{y}$, $\bar{y}+\bar{z}$, $\bar{z}+\bar{x}$ are linearly independent vectors.

(C) (1) Text page No. 175.

(2) Here the direction of vectors $\overrightarrow{AP}$ and $\overrightarrow{PB}$ are same and $\frac{AP}{PB} = \frac{m}{n}$. Hence $n\overrightarrow{AP} = m\overrightarrow{PB}$



$\therefore n(\overrightarrow{OP} - \overrightarrow{OA}) = m(\overrightarrow{OB} - \overrightarrow{OP})$

$\therefore (m+n)\,\overrightarrow{OP} = n(\overrightarrow{OA}) + m(\overrightarrow{OB})$.

(D) (1) Here the velocity of the boat is

$\bar{u} = 0\cdot\bar{i} + 6\sqrt{2}\bar{j}$

$\therefore \bar{u} = \frac{12}{\sqrt{2}}\bar{j}$

Suppose the true velocity of the wind is $\bar{v}$.

The wind blows from the south-east.

i.e. it seems to go in the direction North-west and the velocity of the wind relative to the boat is $\bar{v} - \bar{u} = 5\cos 135°\bar{i} + 5\sin 135\bar{j}$

$\therefore \bar{v} - \bar{u} = -\frac{5}{\sqrt{2}}\bar{i} - \frac{5}{\sqrt{2}}\bar{j}$.

Now the true velocity of wind $\bar{v} = (\bar{v} - \bar{u}) + \bar{u}$

$\therefore \bar{v} = -\frac{5}{\sqrt{2}}\bar{i} + \frac{17}{\sqrt{2}}\bar{j}$. Now $|\bar{v}| = \sqrt{\frac{25}{2} + \frac{289}{2}} = \sqrt{157}$ units

and if $\bar{v}$ makes an angle θ with $\overrightarrow{OX}$, then

$\cos\theta = \frac{\bar{v}\cdot\bar{i}}{|\bar{v}||\bar{i}|} = -\frac{5}{\sqrt{2}\sqrt{157}\times 1} = -\frac{5}{\sqrt{314}}$

$\therefore \theta = \pi - \cos^{-1}\frac{5}{\sqrt{314}}$ with the East towards to the North.

13

(2) Here $(2a, a, 4) \perp (a, -2, -1)$

$\therefore (2a, a, 4) \cdot (a, -2, -1) = 0$

$\therefore 2a^2 - 2a - 4 = 0$

$\therefore a^2 - a - 2 = 0$

$\therefore (a-2)(a+1) = 0$

$\therefore a = 2$ or $a = -1$.

A.5.(A)(1) Text page No. 191.

(2) Text page No. 204.

OR

(2) Text page No. 203.

(B) (1) Text page 214.

(2) Here $l + m + n = 0$ — (1)

$l^2 - m^2 + n^2 = 0$ — (2)

By (1), $m = -(l+n)$ substituting in (2).

$l^2 - (l+n)^2 + n^2 = 0$

$\therefore -2ln = 0$

$\therefore l = 0$ or $n = 0$

(i) If $l = 0$ then from (1), $m = -n$ so we get direction ratio of the first diagonal vector $0, -n$ and n.

(ii) If $n = 0$ then from the equation (2) $m = -l$ and so direction ratio of the second diagonal vector is $l, -l, 0$.

$\therefore$ For the measure of angle θ between two diagonal vectors. is,

$$\cos\theta = \frac{(0, -n, n) \cdot (l, -l, 0)}{\sqrt{2n^2} \cdot \sqrt{2l^2}}$$

$$= \frac{0 + nl + 0}{2 \cdot nl}$$

$\therefore \cos\theta = \frac{1}{2}$ $\quad (\because 0 < \theta < \pi/2)$.

$\therefore \theta = \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$

OR

Here vector forms of given lines in R^3 is $\bar{r} = (3, -15, 9) + k(2, -7, 5)$ and $\bar{r} = (-1, 1, 9) + k(2, 1, -3)$ $k \in R$ comparing with the vector equations $\bar{r} = \bar{a} + k\bar{l}$, $\bar{r} = \bar{b} + k\bar{m}$, $k \in R$, we get $\bar{a} = (3, -15, 9)$, $\bar{b} = (-1, 1, 9)$ $\bar{l} = (2, -7, 5)$ and $\bar{m} = (2, 1, -3)$.

$\therefore \bar{l} \times \bar{m} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix} = \bar{i}(21-5) - \bar{j}(-6-10) + \bar{k}(2+14)$. $\underline{14}$.

$= (16, 16, 16)$.

$\therefore |\bar{l} \times \bar{m}| = \sqrt{(16)^2+(16)^2+(16)^2} = 16\sqrt{3}$.

Now $\bar{u} = \frac{\bar{l} \times \bar{m}}{|\bar{l} \times \bar{m}|} = \frac{1}{\sqrt{3}}(1, 1, 1)$.

and $\bar{a} - \bar{b} = (3, -15, 9) - (-1, 1, 9)$

$= (4, -16, 0)$.

$\therefore$ The perpendicular distance between two lines is $|(\bar{a}-\bar{b}) \cdot \bar{u}| = |(4, -16, 0) \cdot \frac{1}{\sqrt{3}}(1, 1, 1)|$

$= \frac{1}{\sqrt{3}} |4 - 16 - 0|$

$= \frac{12}{\sqrt{3}}$

$= 4\sqrt{3}$

$\therefore$ Thus, the shortest distance between two lines is $4\sqrt{3}$ units.

(c) (1) Here suppose $V(4, 5, 1)$, $A(0, -1, -1)$, $B(3, 9, 4)$ and $C(-4, 4, 4)$.

$\therefore \vec{VA} = (-4, -6, -2)$, $\vec{VB} = (-1, 4, 3)$ and $\vec{VC} = (-8, -1, 3)$.

Now $\vec{VA} \cdot (\vec{VB} \times \vec{VC}) = [\vec{VA}\ \vec{VB}\ \vec{VC}]$

$= \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = -60 + 126 - 66$.

$= 0$

$\therefore \vec{VA}$, $\vec{VB}$ and $\vec{VC}$ are collinear.

$\therefore$ The points V, A, B, C are coplaner.

$\therefore$ They cannot be the vertices of any tetrahedron.

(2) Suppose equation of desired sphere through $(0, 0, 0)$, $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ is

$S: x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ — (1)

As $(0, 0, 0) \in S \quad \therefore \quad d = 0$

$(a, 0, 0) \in S \quad \therefore \quad a^2 + 2ua = 0 \Rightarrow u = -\frac{a}{2}$

$(0, b, 0) \in S \quad \therefore \quad b^2 + 2vb = 0 \Rightarrow v = -\frac{b}{2}$

$(0, 0, c) \in S \quad \therefore \quad c^2 + 2wc = 0 \Rightarrow w = -\frac{c}{2}$.

$\therefore$ The equation of the sphere will be

$x^2 + y^2 + z^2 - ax - by - cz = 0$

$\therefore$ Centre $(-u, -v, -w) = (a/2, b/2, c/2)$

& radius $r = \sqrt{u^2+v^2+w^2-d} = \frac{1}{2}\sqrt{a^2+b^2+c^2}$.

15.

(D) (D) Here from equation of given lines

$\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$

Comparing with $\bar{r} = \bar{a} + k\bar{l}$, $\bar{r} = \bar{b} + k\bar{m}$, $k \in R$

$\bar{a} = (4, -3, -1)$, $\bar{b} = (1, -1, -10)$, $\bar{l} = (1, -4, 7)$

and $\bar{m} = (2, -3, 8)$.

Now $\bar{l} \times \bar{m} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -4 & 7 \\ 2 & -3 & 8 \end{vmatrix} = \bar{i}(-11) - \bar{j}(-6) + \bar{k}(5)$

$\bar{l} \times \bar{m} = (-11, +6, 5)$

& $\bar{b} - \bar{a} = (1, -1, -10) - (4, -3, -1)$

$= (-3, 2, -9)$

$\therefore (\bar{b} - \bar{a}) \cdot (\bar{l} \times \bar{m}) = (-3, 2, -9) \cdot (-11, 6, 5)$

$= 33 + 12 - 45$

$= 0.$

$\therefore$ Both the lines are intersecting lines.

Now equation of plane passing through such lines according to $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$ is

$\begin{vmatrix} x-4 & y+3 & z+1 \\ 1 & -4 & 7 \\ 2 & -3 & 8 \end{vmatrix} = 0$

$\therefore (x-4)(-32+21) - (y+3)(8-14) + (z+1)(-3+8) = 0$

$\therefore -11x + 44 + 6y + 18 + 5z + 5 = 0$

$\therefore 11x - 6y - 5z = 67.$

$\boxed{\text{OR}}$

Here the planes are $x + y + 2z = 4$ and $2x - y + z = -1$

i.e. $(x, y, z) \cdot (1, 1, 2) = 4$

& $(x, y, z) \cdot (2, -1, 1) = -1$

$\therefore$ Comparing with $\bar{r} \cdot \bar{n} = d$ their normal vectors are $\bar{n}_1 = (1, 1, 2)$, $\bar{n}_2 = (2, -1, 1)$.

Now $\bar{n} = \bar{n}_1 \times \bar{n}_2 = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix} = 3\bar{i} + 3\bar{j} - 3\bar{k}$

$\therefore \bar{n} = (3, 3, -3).$

Now for a common point of intersecting plane taking $z = 0$. $x + y = 4$ and $2x - y = -1$

Solving these equations $x = 1$ and $y = 3$

Thus, one common point of intersecting planes is $\bar{a}(1, 3, 0)$. Also the equation of the line of both the intersecting planes is $\bar{r} = \bar{a} + k\bar{n}$, $k \in R$

$\bar{r} = (1, 3, 0) + k'(3, 3, -3)$; $k' \in R$

$\therefore \bar{r} = (1, 3, 0) + k(1, 1, -1)$. where $k = 3k' \in R$.

Thus, the equation of the intersecting line of the planes is

$\bar{r} = (1, 3, 0) + k(1, 1, -1)$, $k \in R$.