MATHS-II (051 E) Q. PAPER SET-I

TOTAL MARKS-75

TIME: 3-Hour

Instructions:

(1) This question paper contains 5 questions.

(2) All are compulsory

(3) Digit to right side indicate the marks of the questions.

Q-I (A) (1) Define a continuous function. Prove that sine function is continuous every where in $\mathbb{R}$. —(2)

(2) Define limit of sequence. Prove that: $\lim_{n\to\infty} \frac{1}{n} = 0$. — (2)

— (4)

(B) Attempt any TWO:

(1) $\lim_{x\to\sqrt{2}} \frac{\pi - 4\cdot\sec^{-1}x}{x-\sqrt{2}}$

(2) $\lim_{x\to 1} \frac{x^{n+1}-(n+1)x+n}{(x-1)^2}$; $n \in \mathbb{N}$.

(3) $\lim_{x\to 0} \frac{\log(1+x)-\log(1-x)}{x}$

(C) (1) Find: $\lim_{x\to -2} \frac{3x^2+5x-2}{3\sin(x+2)+2x+x^2}$ - (2)

(2) Answer any TWO: - (2)

(i) $\lim_{x\to\infty} x\left(\sqrt[x]{\pi/4} - 1\right)$

(ii) If $f(x) = 2x+3$; $x<1$

$= 5$; $x=1$

$= x^2+4$; $x>1$ then examine the continuity of f at $x=1$.

(iii) $\lim_{x\to 2} \frac{(\cos\alpha)^x+(\sin\alpha)^x-1}{x-2}$; $\cos\alpha>0, \sin\alpha>0$ α - constant

(D) (1) If $x = e^{\tan^{-1}\left(\frac{y-x^2}{x^2}\right)}$ then find $\frac{dy}{dx}$ —(2)

(2) If $y = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\cdots$ then prove that $\frac{dy}{dx}=y$ —(1)

Q.2(A)(1) State and prove multiplication rule of derivative of ax^n — (2)

(2) P.T log function is differentiable and P.T $\frac{d}{dx}(\log_e x) = \frac{1}{x}$ $(x \in R^+)$ — (1)

(3) State mean value theorem — (1)

Q.2(B)(1) Answer any two — (4)

(1) If $y = \sin(\cos(\sin(e^x+1)))$ find $\frac{dy}{dx}$

(2) If $y = \cos^{-1}(4x^3 - 3x)$ find $\frac{dy}{dx}$ if $\frac{1}{2} < x < 1$

(3) If $y = \sin(m\sin^{-1}x)$ then P.T $(1-x^2)y_2 - xy_1 + m^2y = 0$

Q.2(C) Answer any two: — (4)

(1) In the calculation of the area of a triangle from the formula $\Delta = \frac{1}{2}bc\sin A$, A was taken as $\frac{\pi}{6}$. Actually an x% error crept into this measure of A. If b, c are constants what is the percentage error in the calculation of area.

(2) P.T if $\lambda_1 \neq \lambda_2$ then the curves $\frac{x^2}{a^2+\lambda_1} + \frac{y^2}{b^2+\lambda_1} = 1$ and $\frac{x^2}{a^2+\lambda_2} + \frac{y^2}{b^2+\lambda_2} = 1$ intersect each other orthogonally.

(3) P.T if $x>0$ then $\frac{\log(1+x)}{x}$ is a decreasing function.

(4) By cutting equal squares from four corners of a 16×10 tin sheet a box is to be constructed. What should be the length of each square if the volume of the box is maximum.

Q.2(D)(1) Find the approximate value of $\sin^{-1}(0.49)$ — (1)

(2) Apply mean value theorem to $y = \log x$ in the interval [1, 2] and find c — (1)

(3) P.T $f(x) = \tan x - x$ is an increasing function $\forall x \in (0, \pi/2)$ — (1)

Q.3(A)(1) Prove that if functions f and g are integrable over interval $I \subset R$, then $\int[f(x)+g(x)]dx = \int f(x)dx + \int g(x)dx$ — (2)

Or

(1) State and prove the method of substitution of integration.

(2) If the function f is continuous on $[0, 2a]$ then prove that (3)

$$\int_0^{2a} f(x)\,dx = \int_0^a f(x)dx + \int_0^a f(2a-x)\,dx$$ — (2)

Q.3 (B) Calculate any two :- — (4)

1) $\int \frac{1+x}{(2+x)^2} e^x dx$

2) $\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx, (x>0)$

3) $\int \log(x+\sqrt{x^2+a^2})\,dx$

Q.3 (C) Calculate any two — (4)

1) $\int_1^2 \frac{x^2+1}{x^4+1}\,dx$

2) $\int_2^3 5^x\,dx$ (as a limit of sum)

3) $\int_0^{\pi/2} \sin^6 x\,dx$

Q.3 (D) (1) If $f(x)>0$ and f, f' are continuous functions and — (1)
$f'(x) \neq 0$, then find

$$\int (f(x))^n f'(x)\,dx = \frac{(f(x))^{n+1}}{n+1} + c \;, n \neq 1.$$

(2) Solve : $\int \frac{x^2}{(x\sin x + \cos x)^2}\,dx$ — (2)

Q.4 (A) Attempt any two : — (4)

(1) Find the general solution of $2\frac{dy}{dx} - y = e^{x/2}$

(2) Find the general solution of $(2x+y)dx - (4x+2y-1)dy = 0$

(3) Obtain the differential of a simple harmonic motion. $y = a\sin\omega t + b\cos\omega t$ where a, b are arbitary constants.

(4) A body having mass 60 kg slides on the top of a table under a force of $54\sin 2t$ newtons. Force of friction is 60 times its velocity and the initial velocity is zero. Express velocity of the body as a function of time

Q.4 (B) (1) Find the volume of the solid generated when the region bounded by $y=x^2$ and $y=4x-x^2$ is rotated about the x-axis

(2) Find the area of the region bounded by the curves $5x^2=y$ and $2x^2-y+9=0$

3) Line $x=c$ divides the area of the region bounded by $y^2=4x$ and $x=16$ into two regions having equal area. Find c

Q.4 (C) Answer any two: — (4)

1) The mean and standard deviation of a random variable X are 10 and 5 respectively. Find $E(x^2)$, $E[x(x+1)]$, $E\left(\frac{x-10}{5}\right)$ and $E\left[\frac{x-10}{5}\right]^2$

2) Nine balanced coins are tossed together once. Find the probability of getting (i) four and (ii) at least six heads.

3) Ramesh participates in a shooting competition. The probability of him shooting a target is 0.2. What is the probability of shooting the target exactly three times out of five trials.

Q.4 (D) (1) Define a solution of a differential equation — (1)

(2) Write the degree and order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^4}$ — (1)

(3) Find:- $\int \frac{(3+\log x)^3}{x}dx$ — (1)

Q.5 (A) 1) Prove that (i) $P(\phi)=0$ (ii) $0 \leq P(A) \leq 1$. $A \in S$ — (2)

OR

1) State and prove addition law of probability for three events.

2) If A and B are independent events then prove that A and B', A' and B' are independent events. — (2)

Q.5 (B) Attempt any Two — (4)

1) 5 men and 5 women are seated at random in a row. Find the probability of the event that in a row. (i) 5 women are seated together. (ii) No two women are seated together.

2) For two events A and B, $P(A \cup B)=0.9$, $P(A \cap B')=0.4$ and $P(B \cap A')=0.3$. Find the value of $P(A)$, $P(B)$ and $P(A' \cup B')$

(5)

3) Suppose that 2 out of 100 men and 2 out of 1000 women in a city suffer from colour blindness. A random selected person of the city is found to be colour-blind. If the person is a man or women is assumed to be equally likely, what is the probability that the selected person is a woman ?

Q.5(C)(1) After selecting two cards randomly from a pack of 52 cards they are put aside. After that from remaining 50 cards one card is selected at random. Find the probability that this card is an ace. —(2)

(2) Find the probability that a woman delivers atleast one male baby during her three pregnancies —(1)

(3) A leap year is taken at random, what is the probability of getting 53 sundays in it. —(1)

Q.5(D) 1) State the meaning of ROM and RAM —(1)

2) Convert $(11011.11)_2$ into decimal and octal form —(2)

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Solution of Maths - II (051-E) ①

PAPER SET - I

Q-I (A) (1) Defn.: Suppose f is a real function of real variable and let $a \in D_f$. If for every $\varepsilon > 0$, we can find $\delta > 0$, such that

$\forall x,\ x \in N(a, \delta),\ x \in D_f \Rightarrow f(x) \in N(f(a), \varepsilon)$

then we say that f is continuous at $x = a$.

Proof: Sine function is continuous.

We must prove that $\forall a \in \mathbb{R}$, $\lim_{x \to a} \sin x = \sin a$.

Now, $|\sin x - \sin a| = \left|2\cos\left(\frac{x+a}{2}\right) \cdot \sin\left(\frac{x-a}{2}\right)\right|$

$\leq \left|2 \sin\left(\frac{x-a}{2}\right)\right| \quad \left(\because \left|\cos\left(\frac{x+a}{2}\right)\right| \leq 1\right)$

$< 2\left|\frac{x-a}{2}\right| \quad (0 < |x-a| < \pi)$

$\therefore |\sin x - \sin a| < |x-a|$

Now, taking $\delta = \varepsilon > 0$; $\varepsilon < \pi$

$0 < |x-a| < \delta \Rightarrow |\sin x - \sin a| < |x-a| < \varepsilon$

$\therefore \lim_{x \to a} \sin x = \sin a.$

(And we may assume that $\varepsilon < \pi$)

(2) Defn.: Suppose $\{a_n\}$ is a sequence and l is a real number. If for each $\varepsilon > 0$ we can find $m \in \mathbb{N}$ such that,

$n \geq m,\ n \in \mathbb{N},\ m \in \mathbb{N} \Rightarrow |a_n - l| < \varepsilon$

then we say that the limit of the seq. $\{a_n\}$ is l and we write $\lim_{n \to \infty} a_n = l$.

②

$\lim_{n\to\infty} \frac{1}{n} = 0$:

Let $\varepsilon > 0$ be given, Now $\left|\frac{1}{n} - 0\right| < \varepsilon$

$\iff \frac{1}{n} < \varepsilon$

$\iff n > \frac{1}{\varepsilon}$

So let $m = \left[\frac{1}{\varepsilon} + 1\right]$ then $m \in \mathbb{N}$ and $m > \frac{1}{\varepsilon}$

Hence given $\varepsilon > 0$, we can find $m \in \mathbb{N}$ such that

$\forall n,\ n \geq m,\ n \in \mathbb{N} \Rightarrow \left|\frac{1}{n} - 0\right| < \varepsilon$

So, $\lim_{n\to\infty} \frac{1}{n} = 0$.

(B) (1) $\lim_{x\to\sqrt{2}} \frac{\pi - 4\cdot\sec^{-1}x}{x-\sqrt{2}}$

$= \lim_{x\to\sqrt{2}} \frac{4\left\{\frac{\pi}{4} - \sec^{-1}x\right\}}{x-\sqrt{2}}$

Let $\sec^{-1}x = t$ $\quad \therefore x = \sec t$

As $x \to \sqrt{2}$, $t \to \pi/4$

$= \lim_{t\to\pi/4} \frac{4\left\{\frac{\pi}{4} - t\right\}}{\sec t - \sqrt{2}} \times \frac{(\sec t + \sqrt{2})}{(\sec t + \sqrt{2})}$

$= 4 \lim_{t\to\pi/4} \frac{(\pi/4 - t)}{\sec^2 t - 2} \times (\sec t + \sqrt{2})$

$= 4\cdot \lim_{t\to\pi/4} \frac{(\pi/4 - t)\cdot\cos^2 t\cdot(\sec t + \sqrt{2})}{1 - 2\cos^2 t}$

$= 4\cdot \lim_{t\to\pi/4} \frac{(\frac{\pi}{4} - t)\cdot\cos^2 t\cdot(\sec t + \sqrt{2})}{(-\cos 2t)}$

$= -4\cdot \lim_{t\to\pi/4} \frac{(\pi/4 - t)\cdot\cos^2 t\cdot(\sec t + \sqrt{2})}{\sin(\pi/2 - 2t)}$

$= -2\cdot \lim_{t\to\pi/4} \frac{(\pi/2 - 2t)}{\sin(\pi/2 - 2t)}\cdot(\cos^2 t)\cdot(\sec t + \sqrt{2})$

$= -2\ (1)\cdot\left(\frac{1}{\sqrt{2}}\right)^2\cdot(\sqrt{2} + \sqrt{2})$ $\quad \left(\because \lim_{\theta\to 0} \frac{\theta}{\sin\theta} = 1\right)$

$= -2\sqrt{2}$.

(2) $\lim_{x \to 1} \dfrac{x^{n+1} - (n+1)x + n}{(x-1)^2}$ ③

Let $x - 1 = h \quad \therefore \quad x = 1 + h$

As $x \to 1, \ h \to 0$

$= \lim_{h \to 0} \frac{1}{h^2} \left\{ (1+h)^{n+1} - (n+1)(1+h) + n \right\}$

$= \lim_{h \to 0} \frac{1}{h^2} \left\{ \binom{n+1}{0} + \binom{n+1}{1} \cdot h + \binom{n+1}{2} \cdot h^2 + \dots + h^{n+1} - n - nh - 1 - h + n \right\}$

$= \lim_{h \to 0} \frac{1}{h^2} \left\{ 1 + (n+1)h + \binom{n+1}{2} \cdot h^2 + \dots + h^{n+1} - nh - 1 - h \right\}$

$= \lim_{h \to 0} \frac{1}{h^2} \left\{ \binom{n+1}{2} h^2 + \binom{n+1}{3} h^3 + \dots + h^{n+1} \right\}$

$= \lim_{h \to 0} \left\{ \binom{n+1}{2} + \binom{n+1}{3} h + \dots + h^{n-1} \right\}$

$= \binom{n+1}{2} + 0 + 0 + \dots + 0$

$\binom{n+1}{2}$

$= \frac{n}{2}(n+1)$

(3) $\lim_{x \to 0} \dfrac{\log(1+x) - \log(1-x)}{x}$

$= \lim_{x \to 0} \frac{1}{x} \cdot \log(1+x) - \lim_{x \to 0} \frac{1}{x} \cdot \log(1-x)$

$= \log\left(\lim_{x \to 0} (1+x)^{1/x} \right) + \log\left(\lim_{x \to 0} (1-x)^{-1/x} \right)$ ($\because$ Limit of comp. fun.)

$= \log_e e + \log_e e$

$= 1 + 1 = 2.$

(C) (1) $\lim_{x \to -2} \dfrac{3x^2 + 5x - 2}{3 \sin(x+2) + 2x + x^2}$

$= \lim_{x \to -2} \dfrac{(x+2)(3x-1)}{3 \sin(x+2) + x(2+x)}$

$= \lim_{x \to -2} \dfrac{(3x-1)}{3\left(\frac{\sin(x+2)}{(x+2)}\right) + x}$ ($\because (x+2) \neq 0$ As $x \to -2$.)

$= \dfrac{3(-2) - 1}{3(1) + (-2)} = \dfrac{-7}{1} = -7$

(4)

(2) (i) $\lim_{x \to \infty} x \left\{ \sqrt[x]{\pi/4} - 1 \right\}$

$= \lim_{x \to \infty} \dfrac{\left(\frac{\pi}{4}\right)^{1/x} - 1}{\left(\frac{1}{x}\right)}$

$= \lim_{\left(\frac{1}{x}\right) \to 0} \dfrac{\left(\pi/4\right)^{1/x} - 1}{\left(1/x\right)}$

$= \log\left(\pi/4\right) \quad \left(\because \lim_{h \to 0} \frac{a^h - 1}{h} = \log_e a\right)$

(ii) $f(x) = 2x + 3 \;;\; x < 1$
$\quad = 5 \;;\; x = 1$
$\quad = x^2 + 4 \;;\; x > 1$

Here, $f(1) = 5 \quad \ldots \ldots (i)$

$\underline{\lim_{x \to 1} f(x):}$

$LHL = \lim_{x \to 1_-} f(x) = \lim_{x \to -1} (2x + 3) = 2(1) + 3 = 5$

$RHL = \lim_{x \to 1_+} f(x) = \lim_{x \to 1+} (x^2 + 4) = 1^2 + 4 = 5$

Here, $LHL = RHL = 5$

$\therefore \lim_{x \to 1} f(x) = 5 \quad \ldots \ldots (ii)$

From (i) & (ii) $\lim_{x \to 1} f(x) = f(1) = 5.$

$\therefore$ f is continuous at $x = 1$.

(iii) $\lim_{x \to 2} \dfrac{(\cos\alpha)^x + (\sin\alpha)^x - (\cos^2\alpha + \sin^2\alpha)}{x - 2}$

$= \lim_{x \to 2} \cos^2\alpha \left\{ \dfrac{(\cos\alpha)^{x-2} - 1}{x - 2} \right\} + \sin^2\alpha \left\{ \dfrac{(\sin\alpha)^{x-2} - 1}{x - 2} \right\}$

$= \cos^2\alpha \cdot \log(\cos\alpha) + \sin^2\alpha \cdot \log(\sin\alpha) \quad \left(\because \lim_{h \to 0} \frac{a^h - 1}{h} = \log_e a\right)$

(Here, $\cos\alpha > 0$ & $\sin\alpha > 0$)

(D) (1) $x = e^{\tan^{-1}\left(\frac{y - x^2}{x^2}\right)}$

$\Rightarrow \log x = \tan^{-1}\left(\frac{y}{x^2} - 1\right)$

$\Rightarrow \tan(\log x) = \frac{y}{x^2} - 1$

$\Rightarrow y = x^2 (1 + \tan(\log x))$

$\Rightarrow y = x^2 + x^2 \cdot \tan(\log x)$

$y = x^2 + x^2 \cdot \tan(\log x)$

(5)

Now, differentiating w.r.t 'x'

$\frac{dy}{dx} = 2x + x^2 \cdot \frac{\sec^2(\log x)}{x} + \tan(\log x) \cdot 2x$

$= 2x(1 + \tan(\log x)) + x \cdot \sec^2(\log x)$

$= x\{2 + 2\tan(\log x) + \sec^2(\log x)\}$

(2) $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\cdots \infty$

Diff. w.r.t. 'x'

$\frac{dy}{dx} = 0 + 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \cdots\cdots$

$= 1 + x + \frac{x^2}{2!} + \cdots\cdots$

$= y$

Q=2 (A) (1) Statement: If $f, g : (a,b) \to \mathbb{R}$ are both differentiable at x and

$\frac{d}{dx}(f(x) \cdot g(x)) = f(x) \cdot \frac{d}{dx}(g(x)) + g(x) \cdot \frac{d}{dx}(f(x))$.

Proof:

$\frac{d}{dx}(f(x) \cdot g(x)) = \lim_{t \to x} \frac{f(t) \cdot g(t) - f(x) \cdot g(x)}{t - x}$

$= \lim_{t \to x} \frac{f(t) \cdot g(t) - f(t) \cdot g(x) + f(t) \cdot g(x) - f(x) \cdot g(x)}{t - x}$

Now, f, g are diff at x, g(x) is constant and $\lim_{t \to x} f(t)$ exists. So by the working rules of limits

$= \lim_{t \to x} f(t) \cdot \lim_{t \to x} \frac{g(t) - g(x)}{t - x} + g(x) \cdot \lim_{t \to x} \frac{f(t) - f(x)}{t - x}$

$= f(x) \cdot \frac{d}{dx}(g(x)) + g(x) \cdot \frac{d}{dx}(f(x))$

($\because$ f is diff at x, so it is conti at x) and so, $\lim_{t \to x} f(t) = f(x)$.

Thus, $\frac{d}{dx}(f(x) \cdot g(x)) = f(x) \cdot g'(x) + f'(x) \cdot g(x)$.

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(2) Let $f(x) = \log_e x$

Now, $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\log_e(x+h) - \log_e x}{h}$

$= \lim_{h \to 0} \frac{1}{h} \cdot \log\left(\frac{x+h}{x}\right)$

$= \lim_{h \to 0} \frac{1}{h} \log\left(1 + h/x\right)$

$= \frac{1}{x} \log\left(\lim_{h \to 0} \log\left(1 + h/x\right)^{x/h}\right)$

$= \frac{1}{x} \cdot \log_e e \quad \left(\because \lim_{x \to 0} (1+x)^{1/x} = e\right)$

$= 1/x$

Thus, $\log_e x$ is differentiable function

and $\frac{d}{dx}(\log_e x) = 1/x$.

(3) Mean Value Theorem:

If f is continuous in $[a, b]$ and differentiable in (a, b) then

we can ~~say~~ ~~that~~ ~~we~~ always find $c \in (a, b)$ such that $\frac{f(b) - f(a)}{b-a} = f'(c)$.

(B) (1) $y = \sin\{\cos(\sin(e^x+1))\}$

Diff w.r.t. 'x'

$\frac{dy}{dx} = \cos\{\cos(\sin(e^x+1))\} \cdot \frac{d}{dx} \cos(\sin(e^x+1))$

$= \cos\{\cos(\sin(e^x+1))\} \left(-\sin(\sin(e^x+1))\right) \cdot \frac{d}{dx}\left(\sin(e^x+1)\right)$

$= -\cos\{\cos(\sin(e^x+1))\} \cdot \{\sin(\sin(e^x+1))\} \cos(e^x+1) \cdot e^x$

(2) $y = \cos^{-1}(4x^3 - 3x)$; $\frac{1}{2} < x < 1$

Let $x = \cos\theta$ OR $\cos^{-1} x = \theta$; $\theta \in [0, \pi]$

Now, $\frac{1}{2} < x < 1 \Rightarrow \cos \pi/3 < \cos\theta < \cos 0$

$\Rightarrow 0 < \theta < \pi/3$ ($\because$ cos fun is $\downarrow$ in 1st quadt)

$\Rightarrow 0 < 3\theta < \pi$ ⑦

Now, $y = \cos^{-1}(\cos 3\theta)$

$= 3\theta \quad (\because 0 < 3\theta < \pi)$

$\therefore \quad y = 3 \cdot \cos^{-1} x$

Now, diff w.r.t. 'x'

$$\frac{dy}{dx} = \frac{-3}{\sqrt{1-x^2}}$$

(3) $\quad y = \sin(m \cdot \sin^{-1} x)$

Diff w.r.t. 'x'

$$\frac{dy}{dx} = \cos(m \cdot \sin^{-1} x) \cdot \frac{d}{dx}(m \cdot \sin^{-1} x)$$

$$\therefore y_1 = \cos(m \cdot \sin^{-1} x) \cdot \frac{m}{\sqrt{1-x^2}}$$

$$\therefore y_1^2 \cdot (1-x^2) = m^2 (1 - \sin^2(m \cdot \sin^{-1} x))$$

$$\therefore y_1^2 \cdot (1-x^2) = m^2 \cdot (1-y^2)$$

Again diff w.r.t. 'x'

$$\therefore (1-x^2) \cdot 2y_1 y_2 + y_1^2 (-2x) = m^2(-2y y_1)$$

$$\therefore 2y_1 \{(1-x^2) \cdot y_2 - x y_1\} = -m^2 y \cdot 2y_1$$

$$\therefore (1-x^2) \cdot y_2 - x y_1 + m^2 y = 0$$ (Here $y \neq$ cosnst fun $\Rightarrow y_1 \neq 0$)

(C) (1) Here, $\Delta = \frac{1}{2} bc \sin A$

$$\therefore \frac{d\Delta}{dA} = \frac{1}{2} bc \cos A$$

$$= \left(\frac{1}{2} bc \sin A\right) \cdot (\cot A)$$

$$= \Delta \cdot \cot A$$

Now, $\left(\frac{d\Delta}{dA}\right)_{A=\pi/6} = \Delta \cdot \cot \pi/6 = \sqrt{3}\Delta.$

Also, $\delta A = x\%$ of $A = \frac{\pi}{6} \cdot \frac{x}{100} = \frac{\pi x}{600}$

Now, error in a area of triangle

$$\delta\Delta \cong \frac{d\Delta}{dA} \cdot \delta A$$

$$= \sqrt{3}\Delta \cdot \frac{\pi x}{600} = \frac{\sqrt{3}\pi x}{6} \cdot \frac{\Delta}{100}$$

$$\therefore \delta\Delta \cong \frac{\sqrt{3}\pi x}{6} \%$$

(2) As the point of intersection, both the equations are satisfied. (8)

$\therefore$ Taking the difference, we get

$$x^2\left(\frac{1}{a^2+\lambda_1}-\frac{1}{a^2+\lambda_2}\right)+y^2\left(\frac{1}{b^2+\lambda_1}-\frac{1}{b^2+\lambda_2}\right)=0$$

$\therefore$ The point of intersection (x, y) also satisfies

$$\frac{x^2(\lambda_2-\lambda_1)}{(a^2+\lambda_1)(a^2+\lambda_2)}+\frac{y^2(\lambda_2-\lambda_1)}{(b^2+\lambda_1)(b^2+\lambda_2)}=0$$

$$\therefore \frac{x^2}{(a^2+\lambda_1)(a^2+\lambda_2)}+\frac{y^2}{(b^2+\lambda_1)(b^2+\lambda_2)}=0 \quad (\because \lambda_1\neq\lambda_2) \quad \text{------ (1)}$$

Now, differentiating the equation of the first curve $\dfrac{2x}{a^2+\lambda_1}+\dfrac{2y}{b^2+\lambda_1}\cdot\dfrac{dy}{dx}=0$

The slope m_1 of the tangent at any point (x, y) on the first curve is given by

$$m_1=-\frac{(b^2+\lambda_1)\,x}{(a^2+\lambda_1)\,y}$$

and similarly the slope m_2 of the tangent at (x, y) to the second curve is given by

$$m_2=-\frac{(b^2+\lambda_2)\,x}{(a^2+\lambda_2)\,y}$$

$\therefore$ The two curves will intersect orthogonally if and only if $m_1\cdot m_2=-1$

Now, $m_1\cdot m_2=-1 \iff \dfrac{(b^2+\lambda_1)(b^2+\lambda_2)}{(a^2+\lambda_1)(a^2+\lambda_2)}=-1$

$$\iff \frac{x^2}{(a^2+\lambda_1)(a^2+\lambda_2)}+\frac{y^2}{(b^2+\lambda_1)(b^2+\lambda_2)}=0$$

which is same as (1).

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(3) Let $f(x) = \dfrac{\log(1+x)}{x}$; $x > 0$

Now, $f'(x) = \dfrac{x \cdot \frac{1}{1+x} - \log(1+x)\cdot 1}{x^2}$

$= \dfrac{1}{x(1+x)} - \dfrac{\log(1+x)}{x^2}$ ... ... (1)

Let $g(x) = \log(1+x) - \dfrac{x}{x+1}$

$g'(x) = \dfrac{1}{1+x} - \left(\dfrac{(1+x) - x}{(1+x)^2}\right)$

$= \dfrac{1}{1+x} - \dfrac{1}{1+x} + \dfrac{x}{(1+x)^2} = \dfrac{x}{(1+x)^2}$ $(\because x > 0)$

$\therefore$ g is an increasing for $\forall x \in R^+$ $(\because g'(x) > 0)$

$\therefore$ g is strictly increasing function.

$x > 0 \Rightarrow g(x) > g(0)$

$\Rightarrow \log(1+x) - \dfrac{x}{1+x} > 0$

$\Rightarrow \log(1+x) > \dfrac{x}{1+x}$

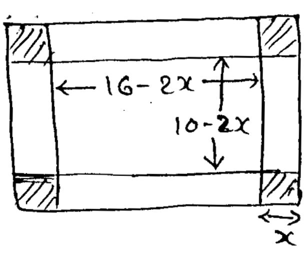
$\Rightarrow \dfrac{\log(1+x)}{x^2} > \dfrac{1}{x(1+x)}$ $(\because$ div by $x^2 > 0)$

... ... (2)

From (1) & (2) $f'(x) < 0$; $x > 0$

$\therefore f(x) = \dfrac{\log(1+x)}{x}$; $x > 0$ is decreasing fun.

(4) Suppose the square of length x unit cut down from four corner of a teen sheet



$\therefore$ The length ~~of the~~ breadth & height of box is $16-2x$, $10-2x$ & x unit resp.

Now, Volume f(x) of a box

$f(x) = (16-2x)(10-2x)x$

$\therefore f(x) = 4x^3 - 52x^2 + 160x$ (10)

Now, $f'(x) = 12x^2 - 104x + 160$

$f''(x) = 24x - 104$

Now, for local maximum or minimum of f

$f'(x) = 0 \Rightarrow (3x-20)(x-2) = 0$

$\Rightarrow x = \frac{20}{3}$ OR $x = 2$

For $x = \frac{20}{3}$, Breadth $= 10 - \frac{40}{3} = -\frac{10}{3} < 0$

$\therefore x \neq 20/3$

For $x = 2$, $f''(2) = 48 - 104 = -56 < 0$.

$\therefore$ At $x = 2$, f is local maximum.

Thus, length of square cutdown from a given teensheet is 2-units.

(D) (1) $\sin^{-1}(0.49)$. Here, $0.49 = 0.50 - 0.01$

$= x + \delta x$

Let $f(x) = \sin^{-1}x$ $\therefore f'(x) = \frac{1}{\sqrt{1-x^2}}$

Now, $f(1/2) = \sin^{-1}(1/2)$ & $f'(\frac{1}{2}) = \frac{1}{\sqrt{1-1/4}}$

$= \pi/6$

$= \frac{2}{\sqrt{3}}$

Now, $f(x+\delta x) \approx f(x) + \delta x \cdot f'(x)$

$= f(1/2) + (-0.01) \cdot \frac{2}{\sqrt{3}}$

$= \frac{\pi}{6} - \frac{1}{50\sqrt{3}}$

$\therefore$ The approximate value of $\sin^{-1}(0.49)$ is

$\frac{\pi}{6} - \frac{1}{50\sqrt{3}}$.

(2) $y = \log x$, $x \in [1, 2]$

Here, $a = 1$ & $b = 2$, $f'(x) = 1/x$

$\frac{f(b) - f(a)}{b-a} = \frac{\log 2 - \log 1}{2-1} = f'(c) = \frac{1}{c}$

$\Rightarrow \log 2 = 1/c \Rightarrow c = \frac{1}{\log 2} = \log_2 e \in (1, 2)$

$(\because 2 < e < 4 \Rightarrow \log_2 2 < \log_2 e < \log_2 2^2 \Rightarrow 1 < \log_2 e < 2$

(3)

$f(x) = \tan x - x$; $x \in (0, \pi/2)$

$f'(x) = \sec^2 x - 1$

$= \tan^2 x > 0$ $(\because x \in (0, \pi/2))$

$\therefore f'(x) > 0$

$\therefore f(x) = \tan x - x$ is increasing function $\forall x, x \in (0, \pi/2)$.

(11)

A.3 (A) (1) $\frac{d}{dx}\left\{\int f(x)dx+\int g(x)dx\right\} = \frac{d}{dx}\int f(x)dx + \frac{d}{dx}\int g(x)dx$ (Rule of derivative)

$= f(x) + g(x)$.

$\therefore \int (f(x)+g(x))dx = \int f(x)dx + \int g(x)dx$. (By def^n of integration).

OR

A3(A) (1) Theory Text book. Chap. 5. Thm. 4.

(2) $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_a^{2a} f(x)dx$; as $0 < a < 2a$.

Let $x = 2a-t$ in second integral on the right hand side : $dx = -dt$, Also $x = a \Rightarrow t = a$

$x = 2a \Rightarrow t = 0$

$\therefore \int_a^{2a} f(x)dx = \int_a^0 f(2a-t)(-dt) = -\int_a^0 f(2a-t)dt = \int_0^a f(2a-t)dt$

$= \int_0^a f(2a-x)dx$.

$\therefore \int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$.

A.3 (B) (1) $I = \int \frac{1+x}{(x+2)^2} e^x dx$

$= \int \left\{\frac{x+2-1}{(x+2)^2}\right\} e^x dx$

$= \int \left\{(x+2)^{-1} + (-(x+2)^{-2})\right\} e^x dx$.

$\left(f(x) = (x+2)^{-1} \Rightarrow f'(x) = -(x+2)^{-2} \right)$

$= e^x (x+2)^{-1} + c$

$\therefore I = \frac{e^x}{x+2} + c$.

(2) $I = \int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx$. Let $x = \tan\theta$, $dx = \sec^2\theta\, d\theta$

$= \int \cos^{-1}(\cos 2\theta) \sec^2\theta\, d\theta$.

$= 2\int \theta \cdot \sec^2\theta\, d\theta$.

$= 2\left[\theta \int \sec^2\theta\, d\theta - \int\left(\frac{d}{d\theta}(\theta)\int \sec^2\theta\, d\theta\right)d\theta\right]$

$= 2\left\{\theta \cdot \tan\theta - \int \tan\theta\, d\theta\right\}$

$= 2\left\{\theta \cdot \tan\theta - \log|\sec\theta|\right\} + c$

$= 2\left\{\theta \cdot \tan\theta - \log|\sqrt{1+x^2}|\right\} + c$.

$\therefore I = 2\left\{x \tan^{-1}x - \log|\sqrt{1+x^2}|\right\} + c$.

$\therefore I = 2x \cdot \tan^{-1}x - \log|1+x^2| + c$.

(3) $I = \int \log(x+\sqrt{x^2+a^2})\,dx$

$= \int \log(x+\sqrt{x^2+a^2}) \cdot 1\, dx$.

$= \log(x+\sqrt{x^2+a^2})\int 1\, dx - \int\left(\frac{d}{dx}(\log(x+\sqrt{x^2+a^2}))\int 1\, dx\right)dx$

$= x\log(x+\sqrt{x^2+a^2}) - \int \frac{1}{(x+\sqrt{x^2+a^2})} \cdot \left(1+\frac{x}{\sqrt{x^2+a^2}}\right) \cdot x\, dx$

$= x\log(x+\sqrt{x^2+a^2}) - \int \frac{x}{\sqrt{x^2+a^2}}\, dx$

(12)

$$I = x\log(x+\sqrt{x^2+a^2}) - \frac{1}{2}\int (x^2+a^2)^{-\frac{1}{2}}\frac{d}{dx}(x^2+a^2)dx.$$

$$= x\log(x+\sqrt{x^2+a^2}) - \frac{1}{2}\frac{\sqrt{x^2+a^2}}{1/2} + c$$

$$\therefore I = x\log(x+\sqrt{x^2+a^2}) - \sqrt{x^2+a^2} + c.$$

A. (3) (c)

① $I = \int_1^2 \frac{x^2+1}{x^4+1}dx$

$$= \int_1^2 \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx$$

$$= \int_1^2 \frac{1+\frac{1}{x^2}}{(x-\frac{1}{x})^2+2}dx.$$

let $x - \frac{1}{x} = t$

$(1+\frac{1}{x^2})dx = dt$

$x = 1 \Rightarrow t = 0$

$x = 2 \Rightarrow t = \frac{3}{2}$

$$= \int_0^{3/2} \frac{dt}{t^2+(\sqrt{2})^2}$$

$$= \frac{1}{\sqrt{2}}\left\{\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)\right\}_0^{3/2}$$

$$= \frac{1}{\sqrt{2}}\left\{\tan^{-1}\frac{3}{2\sqrt{2}} - \tan^{-1}0\right\}.$$

$$\therefore I = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3}{2\sqrt{2}}\right).$$

② $I = \int_2^3 5^x dx.$ $f(x) = 5^x$ is an exponential function so continuous on $[2,3]$. Dividing $[2,3]$ into n-sub. int. of equal length,

$h = \frac{b-a}{n} = \frac{1}{n}.$

$\Leftrightarrow nh = 1.$

$f(a+ih) = f(2+ih)$

$= 5^{2+ih}$

$= 25\cdot 5^{ih}$

As $n\to\infty \Rightarrow h\to 0$

$$= \lim_{h\to 0} h\sum_{i=1}^{n} f(2+ih).$$

$$= \lim_{h\to 0} h\sum_{i=1}^{n} 25\cdot 5^{ih}$$

$$\therefore I = 25\lim_{h\to 0} h\cdot S_n. \quad —①$$

Now $S_n = \sum_{i=1}^{n} 5^{ih}$

$$= \{5^h + 5^{2h} + \dots + 5^{nh}\}$$

$$= \frac{5^h\{(5^h)^n - 1\}}{5^h-1}.$$

$$= \frac{5^h(5-1)}{5^h-1}$$

$$\therefore S_n = \frac{4\cdot 5^n}{5^h-1}$$

Now by ①, $I = 25\lim_{h\to 0} h\cdot S_n$

$$= 25\lim_{h\to 0} h\cdot\left\{\frac{4\cdot 5^n}{5^h-1}\right\}.$$

$$= 100\frac{\{\lim_{h\to 0}5^h\}}{\{\lim_{h\to 0}\frac{5^h-1}{h}\}}.$$

$$\therefore I = \frac{100\times 5^0}{\log_e 5} = 100\log_5 e.$$

(13)

(3) $I = \int_0^{\pi/2} \sin^6 x \, dx$ —(1)

$= \int_0^{\pi/2} \{\sin(\pi/2 - x)\}^6 dx \quad (\because \int_0^a f(x)dx = \int_0^a f(a-x)dx)$

$\therefore I = \int_0^{\pi/2} \cos^6 x \, dx$ —(2)

Adding (1) & (2).

$2I = \int_0^{\pi/2} \sin^6 x + \cos^6 x \, dx$

$= \int_0^{\pi/2} (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x \cdot \cos^2 x \, dx.$

$= \int_0^{\pi/2} 1 - \frac{3}{4} \sin^2 2x \, dx.$

$= \int_0^{\pi/2} 1 - \frac{3}{8} + \frac{3}{8} \cos 2x \, dx.$

$= \int_0^{\pi/2} \frac{5}{8} dx + \frac{3}{8} \int_0^{\pi/2} \cos 2x \, dx$

$2I = \frac{5}{8} [x]_0^{\pi/2} + \frac{3}{8} \left[\frac{\sin 2x}{2}\right]_0^{\pi/2}$

$\therefore 2I = \frac{\pi}{2} \times \frac{5}{8} + \frac{3}{16}(0 - 0).$

$\therefore I = \frac{5\pi}{32}.$

A.3 (D) (1) Theory :-

Let $f(x) = t$ so $\frac{dt}{dx} = f'(x)$

Since $f'(x) \neq 0$ and continuous, $t = f(x)$ is one-one

$\therefore \int [f(x)]^n f'(x) dx = \int t^n dt = \frac{t^{n+1}}{n+1} + C = \frac{[f(x)]^{n+1}}{n+1} + C.$

(2) $I = \int \frac{x^2}{(x\sin x + \cos x)^2} dx.$

$= \int \frac{x^2 \cos x}{(x\sin x + \cos x)^2 \cdot \cos x} dx.$

$= \int \frac{x \cos x}{(x\sin x + \cos x)^2} \cdot \frac{x}{\cos x} dx.$

$= \frac{x}{\cos x} \int (x\sin x + \cos x)^{-2} \frac{d}{dx}(x\sin x + \cos x) dx$

$- \int \frac{d}{dx}\left(\frac{x}{\cos x}\right) \int (x\sin x + \cos x)^{-2} \frac{d}{dx}(x\sin x + \cos x) dx$

$\therefore I = \frac{x}{\cos x} \cdot \frac{(x\sin x + \cos x)^{-1}}{-1} - \int \frac{(\cos x + x\sin x)}{\cos^2 x} \times \left(-\frac{1}{\cos x + x\sin x}\right) dx$

$\therefore I = -\frac{x}{\cos x (x\sin x + \cos x)} + \int \sec^2 x \, dx.$

$\therefore I = -\frac{x}{\cos x \cdot (x\sin x + \cos x)} + \tan x + C.$

$\therefore I = \frac{-x + x\sin^2 x + \cos x \sin x}{\cos x (x\sin x + \cos x)} + C = \frac{(\sin x - x \cdot \cos x)}{x \cdot \sin x + \cos x} + C$

A.4(A) (1) $2\frac{dy}{dx} - y = e^{x/2}$ (14)

$\therefore \frac{dy}{dx} - \frac{1}{2}y = \frac{1}{2}e^{x/2}$

It is of the type $\frac{dy}{dx} + Py = Q$ linear differential equation. $P = -\frac{1}{2}$, $Q = \frac{1}{2}e^{x/2}$

Now multiplying eqn. by $e^{\int P dx} = e^{\int -\frac{1}{2}dx} = e^{-x/2}$,

$e^{-x/2}\frac{dy}{dx} - \frac{1}{2}y e^{-x/2} = \frac{1}{2}e^{x/2}.e^{-x/2}$

$\therefore \frac{d}{dx}(y.e^{-x/2}) = \frac{1}{2}.$

Now integrating,

$\therefore y.e^{-x/2} = \frac{x}{2} + c_1$

$\therefore 2y.e^{-x/2} = x + c.$ $c = 2c_1$ = arbitrary constant.

$\therefore 2y = x.e^{x/2} + c.e^{x/2} = (x+c)e^{x/2}$

(2) $(2x+y)dx - (4x+2y-1)dy = 0$

Here $ab' - a'b = 2(-2) - 4(-1) = 0$

Let $2x+y = z$. Hence $2 + \frac{dy}{dx} = \frac{dz}{dx}$.

Simplifying,

$z - (2z-1)(\frac{dz}{dx} - 1) = 0$

$\therefore z + 4z - 2 - (2z-1)\frac{dz}{dx} = 0$

$\therefore \frac{(2z-1)}{5z-2}dz = dx.$

$\therefore \frac{\frac{2}{5}(5z-1) - \frac{1}{5}}{5z-2}dz = dx$

$\therefore \frac{2}{5}dz - \frac{1}{5(5z-2)}dz = dx$

$\therefore$ Integrating, $\frac{2z}{5} - \frac{1}{25}\log|5z-2| = x - \frac{c}{2}$ (C = arbitrary const.)

Substituting for z,

$\log|10x+5y-2| = 10y - 5x + c'$. $c' = 5c.$

(3) $y = a\sin\omega t + b\cos\omega t$, where a, b are arbitrary constant. Differentiating w.r.t. t twice,

$\frac{dy}{dt} = \omega a\cos\omega t - b\omega\sin\omega t.$

$\therefore \frac{d^2y}{dt^2} = -\omega^2 a\sin\omega t - b\omega^2\cos\omega t.$

$= -\omega^2(a\sin\omega t + b\cos\omega t) = -\omega^2 y.$

$\therefore$ Thus $y_2 + \omega^2 y = 0$ is obtained in which a, b are eliminated.

(15)

(4) Here by Newton's formula,

$F = m \times a$

Weight of body (Kg) $\times$ Acceleration (m/s^2) = Force acting on body

= Applied force

If Velocity V m/sec then $f = \frac{dv}{dt}$ = acceleration, – Force of friction

$\therefore 60 \times \frac{dv}{dt} = 54 \sin 2t - 60V$. $\left(\frac{1}{2}\right)$

$\frac{dv}{dt} = \frac{9}{10} \sin 2t - V$.

$\therefore \frac{dv}{dt} + V = \frac{9}{10} \sin 2t$ — (1)

Comparing with linear diff^{al} eq^n. $\frac{dy}{dx} + Py = Q$.

$P = 1$, $Q = \frac{9}{10} \sin 2t$.

Multiplying (1) by $e^{\int P dt} = e^t$ $\left(\frac{1}{2}\right)$

$e^t \frac{dv}{dt} + Ve^t = \frac{9}{10} \sin 2t \cdot e^t$

$\therefore \frac{d}{dt}(V \cdot e^t) = \frac{9}{10} e^t \sin 2t$.

Here $\int e^{at} \sin bt \, dt = \frac{e^{at}}{a^2+b^2}(a \sin bt - b \cos bt) + c$

$\therefore e^t V = \frac{9}{10} e^t \frac{(\sin 2t - 2\cos 2t)}{1+4} + C$ $\left(\frac{1}{2}\right)$

$\therefore$ Multiplying by e^{-t}

$\therefore V = \frac{9}{50}(\sin 2t - 2\cos 2t) + C \cdot e^{-t}$

Now $t = 0$ then $V = 0$

$0 = \frac{9}{50}(0 - 2(1)) + C$

$C = \frac{9}{25}$

$\therefore$ Required formula for velocity of a body.

$V = \frac{9}{50}(\sin 2t - 2\cos 2t + 2 \cdot e^{-t})$ m/sec. $\left(\frac{1}{2}\right)$

2

A·4(B)

(1) $y = x^2$, $y = 4x - x^2$ (16)

$\therefore x^2 = 4x - x^2$

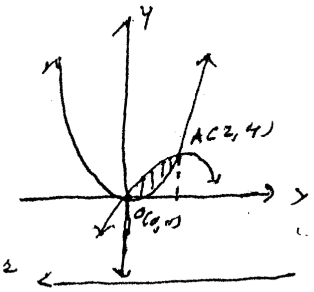
$\therefore 2x^2 - 4x = 0$

$\therefore 2x(x-2) = 0$

$\therefore x = 0$ or $x = 2$

$\therefore y = 0$ or $y = 4$

$O(0,0)$ & $A(2,4)$ are their pts. of intersection.



Now revolving bounded region about x axis volume of a solid generated by this revolution is

$V = \pi \int_a^b (f_1(x))^2 - (f_2(x))^2 dx$, where $f_1(x) = 4x - x^2$, $f_2(x) = x^2$

$= \pi \int_0^2 (4x - x^2)^2 - (x^2)^2 dx$

$= \pi \int_0^2 16x^2 - 8x^3 + x^4 - x^4 dx$

$= 8\pi \int_0^2 2x^2 - x^3 dx$

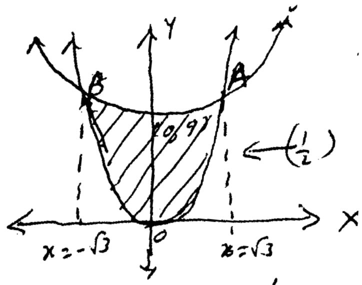
$= 8\pi \left[\frac{2x^3}{3} - \frac{x^4}{4} \right]_0^2$

$= 8\pi \left[\left(\frac{16}{3} - \frac{16}{4} \right) - (0 - 0) \right]$

$= 8\pi \times \frac{16}{12}$

$\therefore V = \frac{32\pi}{3}$ unit.

(2) $5x^2 = y$ $y = 2x^2 + 9$



$\leftarrow (\frac{1}{2})$

$\therefore 5x^2 = 2x^2 + 9$

$x = \pm\sqrt{3}$

$y = 15$

$A(\sqrt{3}, 15)$, $B(-\sqrt{3}, 15)$

Area between two ~~bounded~~ intersecting curves is $A = |I|$. where $f_1(x) = 2x^2 + 9$, $f_2(x) = 5x^2$.

$I = \int_a^b f_1(x) - f_2(x) dx$

$= \int_{-\sqrt{3}}^{\sqrt{3}} 2x^2 + 9 - 5x^2 dx$

$= 2\int_0^{\sqrt{3}} 9 - 3x^2 dx$. ($\because$ Even f^n)

$= 2\left[9x - x^3\right]_0^{\sqrt{3}}$

$= 2\left[9\sqrt{3} - 3\sqrt{3}\right]$

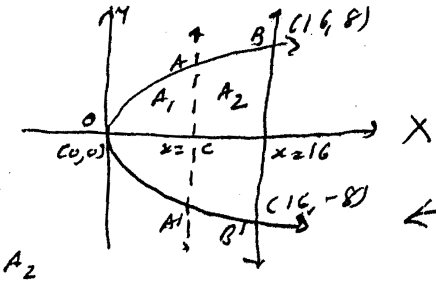
$\therefore I = 12\sqrt{3}$

$\therefore A = |I| = 12\sqrt{3}$ Unit.

(3) $y^2 = 4x$, $x = 16$, $x = c$. (17)

$\therefore y^2 = 64$

$y = \pm 8$



Here line $x = c$ bisect the bounded region formed by $y^2 = 4x$ & $x = 16$. $\leftarrow \frac{1}{2}$

Comparing the region A_1 & A_2 in a upper semi plane to X-axis.

Also $y^2 = 4x$

$\therefore y = 2\sqrt{x}$. ($y > 0$ in I^{st} quadrant) $\leftarrow \frac{1}{2}$

Here $A_1 = A_2$

$\therefore \int_0^c 2\sqrt{x}\,dx = \int_c^{16} 2\sqrt{x}\,dx$.

$\therefore 2\left[\frac{x^{3/2}}{3/2}\right]_0^c = 2\left[\frac{x^{3/2}}{3/2}\right]_c^{16}$ $\leftarrow$ $(\frac{1}{2})$

$\therefore c^{3/2} - 0 = (16)^{3/2} - c^{3/2}$

$\therefore 2 \cdot c^{3/2} = 64$

$\therefore c = (32)^{2/3} = (2^5)^{2/3}$ $\leftarrow$ $(\frac{1}{2})$

$\therefore c = 2^{10/3}$.

[2]

A.4 (C) (1) Mean, $E(X) = 10$, Standard deviation $\sigma_x = \sqrt{V(X)} = 5$.

$\therefore \sigma_X^2 = V(X) = 25$.

Here $V(X) = E(X^2) - (E(X))^2$

$\therefore E(X^2) = V(X) + (E(X))^2$

$= 25 + 100$

$\therefore E(X^2) = 125$ $\leftarrow$ $(\frac{1}{2})$

$E(X(X+1)) = E(X^2 + X)$

$= E(X^2) + E(X)$

$= 125 + 10$

$= 135$ $\leftarrow$ $(\frac{1}{2})$

$E\left(\frac{X-10}{5}\right) = E\left(\frac{1}{5}X - 2\right)$

$= \frac{1}{5}E(X) - 2$

$= \frac{1}{5}(10) - 2$

$\therefore E\left(\frac{X-10}{5}\right) = 0$ $\leftarrow$ $(\frac{1}{2})$

$E\left(\frac{X-10}{5}\right)^2 = E\left(\frac{X^2 - 20X + 100}{25}\right)$

$= \frac{1}{25}E(X^2) - \frac{4}{5}E(X) + 4 = \frac{125}{25} - \frac{4}{5} \times 10 + 4 = 5 - \frac{40}{5} + 4$

$= 9 - 8 = 1 \leftarrow (\frac{1}{2})$

[2]

(18)

(2) Here tossing a coin head occurs on the upper side of coin is taken as success of random experiment then probability for success is $p = \frac{1}{2}$ & Failure is $q = \frac{1}{2}$

Let X denote number of head occuring on tossing 9 balanced coins then X is a binomial random variable with parameters $n=9$, $p=\frac{1}{2}$.

Now probability distribution of X is,

$$P(X=x) = \binom{9}{x}\left(\frac{1}{2}\right)^x \cdot \left(\frac{1}{2}\right)^{9-x} \quad x = 0, 1, 2, 3, \ldots 9.$$

Now, (i) Probability of getting 4 heads.

$$P(X=4) = P(4)$$
$$= \binom{9}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^5$$
$$= \frac{9!}{5!4!} \times \left(\frac{1}{2}\right)^9$$
$$= \frac{9\times8\times7\times6}{24} \times \frac{1}{512}$$
$$= \frac{63}{256}$$

(ii) Probability of getting atleast 6 heads.

$$P(X \geq 6) = P(6) + P(7) + P(8) + P(9)$$
$$= \binom{9}{6}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^3 + \binom{9}{7}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^2 + \binom{9}{8}\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right) + \binom{9}{9}\left(\frac{1}{2}\right)^9$$
$$= \left(\frac{1}{2}\right)^9 \left\{ \binom{9}{6} + \binom{9}{7} + \binom{9}{8} + \binom{9}{9} \right\}.$$
$$= \frac{1}{512} \times (84 + 36 + 9 + 1)$$
$$= \frac{130}{512}$$
$$= \frac{65}{256}$$

(3) Ramesh shooting the target is taken as success then prob. of success is $p = 0.2$

Prob. of failure $= q = 0.8$

Suppose Ramesh shoot the target X times of 5 trials then X is binomial random variable ~~the~~ & parameters are $n = 5$ & $p = 0.2$.

Binomial prob. distribution of X is

$$P(X=x) = \binom{5}{x}(0.2)^x(0.8)^{5-x}, \quad x = 0, 1, 2, 3, 4, 5.$$

Now prob. that Ramesh shooting the target exactly three times is

$$P(X=3) = P(3) = \binom{5}{3}(0.2)^3(0.8)^2 = 10 \times (0.008)(0.64)$$
$$P(X=3) = 0.0512.$$

(19)

A. 4 (D)

(1) Def^n :- Let a differential equation in variables x & y be given. If we can find a function $y = f(x)$ such that x, y and its derivatives identically satisfy the differential equation the function $y = f(x)$ is called a solution of differential equation. ← (1)

(2) $\left(\frac{d^2y}{dx^2}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^4}$

$\therefore \left(\frac{d^2y}{dx^2}\right)^6 = 1 + \left(\frac{dy}{dx}\right)^4$

In a given differential equation heighest order derivative is $\frac{d^2y}{dx^2}$ & heighest degree of its is 6.

$\frac{1}{2} + \frac{1}{2} =$ (1)

$\therefore$ Order = 2 . Degree = 6.

(3) Find : $\int \frac{(3+\log x)^3}{x} dx$ ← $\left(\frac{1}{2}\right)$

$I = \int (3+\log x)^3 \frac{d}{dx}(3+\log x)\, dx$ ← $\left(\frac{1}{2}\right)$

$\therefore I = \frac{1}{4}(3+\log x)^4 + C.$

[1]

A. 5 (A) (i) $P(\phi) = 0$

Events ϕ and U are mutually exclusive ← $\left(\frac{1}{2}\right)$

and $\phi \cup U = U$

Suppose $A_1 = \phi$ and $A_2 = U$

Then $P(\phi \cup U) = P(U) = P(\phi) + P(U)$ ($\because$ axiom 3)

$\therefore 1 = P(\phi) + 1.$

Hence $P(\phi) = 0$ ← $\left(\frac{1}{2}\right)$

[1]

(ii) Clearly $\phi \subset A$ and $A \subset U$

According axiom (1), $P(A) \geq 0$

Since $\phi \subset A$ and $A \subset U$ we have ← (½)

$0 = P(\phi) \leq P(A)$ & $P(A) \leq P(U) = 1$ ($\because$ axiom 2) (Thm. 3)

$\therefore 0 \leq P(A) \leq 1$ ← $\left(\frac{1}{2}\right)$

[01]

OR

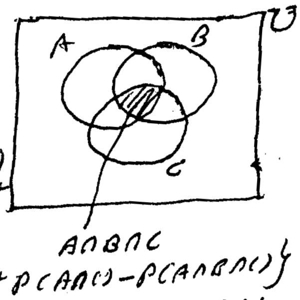
(P. T. O.)

(1) State and prove addition law of probability for three events. (20)

→ If A, B and C are events, then

$$P(A\cup B\cup C) = P(A)+P(B)+P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$$

→ Proof:-



$P(A\cup B\cup C) = P[A\cup(B\cup C)]$

$= P(A) + P(B\cup C) - P(A\cap(B\cup C))$

$= P(A) + P(B) + P(C) - P(B\cap C) - P[(A\cap B)\cup(A\cap C)]$

$(\because$ Thm. 4 and distributive law of set operations)

$= P(A)+P(B)+P(C) - P(B\cap C) - \{P(A\cap B) + P(A\cap C) - P(A\cap B\cap C)\}$

$(\because (A\cap B)\cap(A\cap C) = A\cap B\cap C)$

$\therefore P(A\cup B\cup C) = P(A)+P(B)+P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C).$

(2) Since $A\cap B$ and $A\cap B'$ are mutually exclusive and since $A = (A\cap B)\cup(A\cap B')$

$P(A) = P(A\cap B) + P(A\cap B')$.

Since A and B are independent we have

$P(A\cap B) = P(A)\cdot P(B)$,

$\therefore P(A) = P(A)\cdot P(B) + P(A\cap B')$.

$\therefore P(A\cap B') = P(A)\{1-P(B)\} = P(A)\cdot P(B')$

Hence by definition of independent events A and B' are independent.

According to De-Morgan's laws

$A'\cap B' = (A\cup B)'$

$\therefore P(A'\cap B') = 1 - P(A\cup B)$

$= 1 - \{P(A) + P(B) - P(A\cap B)\}$

$= 1 - P(A) - P(B) + P(A)\cdot P(B)$

$(\because$ A and B are ind.)

$= (1-P(A))(1-P(B))$

Thus $P(A'\cap B') = P(A')\cdot P(B')$.

$\therefore$ Events A' & B' are independent.

A. 5(B) 21

(1) Here 5 men & 5 women that is 10 persons seated in row randomly.

$\therefore n = 10!$

Suppose event A : In a random arrangement of 10 persons in a row 5 women seated together

5 women seat in row in 5! different ways

5 men " " " 5! " ".

Now group of 5 women placed between two men's or at the both end points in 6 ways.

Possible arrangements = r = 5! 5! × 6.

$\therefore P(A) = \frac{r}{n} = \frac{6 \times 5! \, 5!}{10!} = \frac{\cancel{6} \times 1 \times \cancel{2} \times \cancel{3} \times \cancel{4} \times \cancel{5}}{\cancel{10} \times \cancel{9} \times \cancel{8} \times \cancel{7} \times \cancel{6}} = \frac{1}{42}.$

$\therefore P(A) = \frac{1}{42}.$

Suppose event B : No two women seated together.

$P(B) = \frac{r}{n} = \frac{6 \times 5! \, 5!}{10!} = \frac{1}{42}.$

$\therefore P(B) = \frac{1}{42}.$

(2) Here $P(A \cup B) = 0.9$, $P(A \cap B') = 0.4$, $P(B \cap A') = 0.3$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B \cap A') = P(B) + P(A \cap B')$.

$\therefore 0.9 = P(A) + 0.3$, $0.9 = P(B) + 0.4$

$\therefore P(A) = 0.6$, $P(B) = 0.5$

$P(A' \cup B') = P((A \cap B)')$

$= 1 - P(A \cap B)$

$= 1 - \{P(A) + P(B) - P(A \cup B)\}$

$= 1 - \{0.6 + 0.5 - 0.9\}$

$= 1 - 0.2$

$\therefore P(A' \cup B') = 0.8$

(P. T. O.)

③ Suppose B_1 : Randomly ~~sel~~ selected person is man. ㉒

B_2 : " " " is woman.

A : " " " suffer from colourblind.

Here $P(B_1) = P(B_2) = 0.5$

Now $P(A/B_1) = \frac{2}{100} = 0.02$ & $P(A/B_2) = \frac{2}{1000} = 0.002$

If randomly selected person suffering from colour blind then that person is woman is an event B_2 and probability of such event is

$$P(B_2/A) = \frac{P(B_2 \cap A)}{P(A)}$$

$$= \frac{P(B_2)\cdot P(A/B_2)}{P(B_1)\cdot P(A/B_1) + P(B_2)\cdot P(A/B_2)}.$$

$$= \frac{0.5 \times (0.002)}{0.5 \times 0.02 + 0.5 \times 0.002}$$

$$= \frac{0.002}{0.020 + 0.002}$$

$$= \frac{0.002}{0.022}$$

$$= \frac{2}{22}$$

$\therefore P(B_2/A) = \frac{1}{11}$.

C (1) Suppose B_1 : From pack of 52 cards randomly selection of two cards one is of ace.

B_2 : " " " " " " two are of ace.

B_3 : None of the card out of 2 randomly selected card is of ace.

A : From remaining 50 cards randomly selected one card is of ace.

$$P(B_1) = \frac{\binom{4}{1}\binom{48}{1}}{\binom{52}{2}} = \frac{4 \times 48}{52 \times 51} \times 2$$

$$P(B_2) = \frac{\binom{4}{2}}{\binom{52}{2}} = \frac{12}{52 \times 51}$$

$$P(B_3) = \frac{\binom{48}{2}}{\binom{52}{2}} = \frac{48 \times 47}{52 \times 51}$$

Here $P(A) = P(B_1)\cdot P(A/B_1) + P(B_2)\cdot P(A/B_2) + P(B_3)\cdot P(A/B_3)$

$$= \frac{4 \times 48}{52 \times 51} \times 2 \times \frac{3}{50} + \frac{4 \times 3}{52 \times 51} \times \frac{2}{50} + \frac{48 \times 47}{52 \times 51} \times \frac{4}{50}$$

$$= \frac{24 \times 48 + 24 + 48 \times 47 \times 4}{50 \times 51 \times 52}$$

$$= \frac{24 \times 49 + 47 \times 192}{50 \times 51 \times 52} = \frac{24(49 + 47 \times 2)}{50 \times 51 \times 52}$$

$$= \frac{24(49 + 94)}{50 \times 51 \times 52}$$

$$= \frac{24 \times 143}{50 \times 51 \times 52} = \frac{11}{425}$$

(23)

(2. Here $U = \{bbb, bbg, bgb, gbb, ggg, ggb, gbg, bgg\}$,

$n = 8$

A = At least one male baby delivers in her three pregnancies.

$n(A) = 7$

$P(A) = \frac{7}{8}$.

(3) In leap year 366 days

i.e. $7 \times 52 + 2$

U = { (Sun, Mon), (Mon, Tue), (Tue, Wed), (Wed, Thus) (Thus, Fri), (Fri, Sat), (Sat, Sun) }

$\therefore n = 7$

A = In a randomly taken leap year there will be 53 sundays

$n(A) = 2$

$P(A) = \frac{2}{7}$

(D) (1) Rom :- Read Only memory.

Ram :- Random Access memory.

(2) $(11011.11)_2$

$= 2^4 \times 1 + 2^3 \times 1 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 1 + 1 \times 2^{-1} + 1 \times 2^{-2}$

$= 16 + 8 + 0 + 2 + 1 + 0.50 + 0.25$

$= (27.75)_{10}$

$(11011.11)_2$

$= (011\,011.110)_2$

$= (33.6)_8$

$011 = 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 3$

$011 = 3$

$110 = 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 6$

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