General Instructions :

(1) Do neat work and show all calculations.

(2) Write all steps.

1. (a) (1) Define left hand limit and prove that if $\lim\limits_{x \to a}$ f(x) exists then,

$$\lim_{x \to a} kf(x) = k \lim_{x \to a} f(x), K \in R$$ 2

(2) Define limit of Sequence and prove that 2

$$\lim_{n \to \infty} r^n = 0, \quad |r| < 1$$

(b) Attempt any two : 4

(1) $\lim\limits_{x \to a} \dfrac{\tan x - \tan a}{\tan^{-1} x - \tan^{-1} a}$

(2) $\lim\limits_{h \to 0} \dfrac{5^{2h} - 25^{Sinh}}{h - Sinh}$

(3) $\lim\limits_{x \to \frac{\pi}{4}} (Sin2x)^{\tan^2 2x}$

(c) (1) Find A, B if f(x) is continuous on R. 2

f(x) = 2Sin x $\quad x \le -\dfrac{\pi}{2}$

= Asinx + B $\quad -\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$

= Cos x $\quad x \ge \dfrac{\pi}{2}$

(2) Attempt any two : 2

(1) $\lim\limits_{x \to 1} \dfrac{\sqrt[n]{x} + \sqrt[n]{x^2} - 2}{x - 1}$ $\quad$ (2) $\lim\limits_{n \to \infty} \sum\limits_{r=1}^{n} \frac{1}{n} 3^{r/n}$

(3) $\lim\limits_{x \to \pi} \dfrac{\sqrt{2 + Cosx} - 1}{(\pi - x)^2}$

(d) (1) If f(x) = x Sin $\dfrac{1}{x}$ for x ≠ 0 and f(0) = 0, prove that f is continuous but not differentiable at x = 0. 2

(2) Find the derivative at x = 0 for f(x) = $|2x + 1|$ using definition. 1

2. (a) (1) Prove that 2

If both f,g : (a, b) $\to$ R are differentiable at x, and $g(x) \neq 0$ then $\frac{f}{g}$

is differentiable at x and $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{(g(x))^2}$

(2) (1) Write rule for derivative of a composite function. 1

(2) Write mean value theorem and its geometrical interpretation. 1

(b) Attempt any two : 4

(1) Find $\frac{dy}{dx}$ for y = $\left(\sqrt{x}\right)^x + x^{\sqrt{x}}$

(2) If $\cos^{-1}\frac{y}{b} = \log\left(\frac{x}{n}\right)^n$ then prove that $x^2y_2 + xy_1 + n^2y = 0$

(3) Find $\frac{dy}{dx}$ y = $\cos^{-1}x + \cos^{-1}\sqrt{1-x^2}$ $|x|<1$

(c) Attempt any two : 4

(1) The height of a tower was measured from a point 200m away from the tower. The angle of elevation was measured as 30° but the true angle of elevation from that point was 30° 12'. What error must have been committed in the calculation of the height of the tower?

(2) Prove that the portion of any tangent to $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ which lies between the two axes is of constant length. (a > 0)

(3) A window is in the shape of semicircle over a rectangle. If the total perimeter of the window is to be kept constant and if we wish maximum amount of light to pass through the window, prove that the length of the rectangle should be twice its height.

(4) Prove that if x > 0 then $\frac{\log(1+x)}{x}$ is decreasing function.

Find the local extreme values

(d) Answer the following questions : (1) of $f(x) = x + 4/x$, $x \in R - \{0\}$ 3

(2) Determine if Rolle's theorem is applicable and if so, determine c such that $f'(c) = 0$, $f(x) = x^3 - 12x$, $x \in [0, 2\sqrt{3}]$

(3) Prove that $4x^2 + 9y^2 = 45$ and $x^2 - 4y^2 = 5$ intersect orthogonally.

3. (a) State rule of Substitution of Indefinite integral and prove that 2

$\int f(x)dx = F(x)$ then

$$\int f(ax+b)dx = \frac{1}{a}F(ax+b) \text{ where}$$

$f : I \rightarrow R$ is continuous on some interval I $(a \neq 0)$

OR

State rule of Integration by parts of Indefinite Integration and prove that

$$\int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}Sin^{-1}\frac{x}{a} + c \ (a>0)$$

(2) Prove that if f is continuous on [0, 2a] then 2

$$\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx$$

(b) Attempt any two : 4

(1) $\int \frac{x^2}{1+x^4}\,dx$ (3) $\int\sqrt{1+\cos 4x}\,dx$ $\left(\frac{\pi}{4} < x < \frac{\pi}{2}\right)$

(2) $\int x\sqrt{2ax-x^2}\,dx$

(c) Attempt any two : 4

(1) Evaluate $\int_a^b$ Sin x dx as limit of Sum.

(2) $\int_0^{\frac{\pi}{4}} \frac{dx}{a^2Cos^2x - b^2Sin^2x}$, $a > 0, b > 0$

(3) $\int_{-\pi}^{\pi} (x^2 + x)$ Sin5x dx

(d) (1) $\int\sqrt{\frac{Sin(x-a)}{Sin(x+a)}}$ dx 2

(2) Prove that $\int Cotx\,dx = \log|Sinx| + c,\ x \neq n\frac{\pi}{2},\ n \in z$ 1

4. (a) Solve any two : (Solve 1 to 3) 4

(1) $xy(xdy + ydx) = 6y^3dy$ and $x = 2 \Rightarrow y = 1$

(2) $\frac{dy}{dx} = \frac{2x-5y+3}{2x+4y-6}$

(3) $(x+y)^2 \frac{dy}{dx} = 2(x+y)^2 - 3$ (3)

(4) The rate of melting of a piece of ice is proportional to its quantity at any moment. In 30 minutes, half of the ice has melted. Prove that $\frac{1}{8}$ the original quantify will remain after 90 minutes.

(b) Answer any two of the following : 4

(1) The region bounded by $x^2 - y^2 = a^2$ and $x = 2a$ $(a > 0)$ is rotated about X - axis. Find the volume of the solid of revolution .

(2) Prove that the area of the region enclosed by the circle $x^2 + y^2 = 64$ and parabola $y^2 = 12x$ is $\frac{16}{3}\left(4\pi + \sqrt{3}\right)$

(3) The region bounded by $y = 4x - x^2$, $x = 1$ $x = 3$ and X-axis is divided in two parts with equal area by $x = C$. Find C.

(c) Answer any two : 4

(1) Probability distribution of a discrete random variable X is defined below.

$P(x) = 0.2$ for $x = 0$

$= Kx$ for $x = 1, 2$

$= K(6 - x)$ for $x = 3, 4$

$= 0$ otherwise

(i) Find constant K

(ii) Draw graph of p(x)

(iii) Find value of $P(X \geq 3)$

(iv) Find V(X)

(2) Probability of a product of a machine being defected is p. Find the probability that the number of defective product is greater than the number of non-defective products out of 5 products selected at random. If $p = \frac{1}{2}$ obtain probability of the event.

(3) An individual tosses a balanced die five times. If the individual gets 5, 4, 3, 2 and 1 headed he receives Rs. 10, 8, 6, 4 and 2 respectively and loses Rs. 16 if no head is obtained. Find the expected gain of the individual.

(d) (1) Fill in the blank with necessary calculation : 1

$\int 2003^{2003^{2003^x}} \cdot 2003^{2003^x} \cdot 2003^x \, dx =$ __________.

(2) (i) Define degree and order of differential equation. 1

1 (ii) Explain method of solving homogeneous differential equation. 1

5. (a) Define the following : 2

(1) (a) Classical definition of Probability

(b) Additive set function

OR

(1) If A and B are two events, then prove that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and write formula of $P(A \cup B \cup C)$

(2) Suppose B_1 and B_2 are mutually exclusive and exhaustive events. 2

If $P(A/Bi)$ are given i = 1, 2 and $P(A) \neq 0$ then prove that

$$P(Bi/A) = \frac{P(Bi)P(A/Bi)}{P(B_1)P(A/B_1) + P(B_2)P(A/B_2)}, \quad i = 1, 2$$

(b) Answer any two of the following : 4

(1) Write the elements of Sample space associated with an experiment of tossing a balanced win four times . Using the classical definition of probability find the probability of following events :

(i) There is atleast one head.

(ii) There are exactly three heads

(iii) There are three or more heads

(iv) There is no tail.

(2) A die is constructed in such a way that the probability that the integer obtained on its face when tossed is proportional to the square of that integer. Find the probability that an integer on the face of a die is even.

(3) Find the probability of four Sundays in month of April.

(c) (1) Box X contains 4 red and 6 white balls and box Y contains 3 red and 7 white balls. A ball is drawn at random from box X and is transferred to box Y. Now a

ball is drawn at random from box Y. What is the probability that selected ball is white? If the selected ball is found red, find the probability that the ball selected from box X and transferred to box Y is white.

(2) (a) If $P(A) = 0.1$, $P(B) = 0.2$ and $P(A \cup B) = 0.25$ then find $P(B / A')$ and $P(A' / B')$. 1

(b) Explain with reasons why the following allocation of probabilities for events A and B is not possible. 1

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{3}$

(d) (1) 1 kilobyte = __________ bytes. 1

(2) Write binary and Octal form of $(93.625)_{10}$ 2

MATHS: II Solution of paper set II

(051) (E) ①

Que: 1 (a) (1) LEFT HAND LIMIT :-

If f is a real function defined on Some interval (c, a). If for every $\varepsilon > 0$ we can find $\delta > 0$ such that

$x \in (a-\delta, a)$, $x \in D_f \Rightarrow |f(x) - l| < \varepsilon$ then we say that as x tends to a from the left $f(x)$ tends to l or the left hand limit of $f(x)$ is l.

i.e. $\lim_{x \to a_-} f(x) = l$.

Now, First suppose $K \neq 0$ and $\lim_{x \to a} f(x) = l$

Given any $\varepsilon > 0$ then $\frac{\varepsilon}{|K|}$ is also a positive number and corresponding to this positive number $\frac{\varepsilon}{|K|}$, there is $\delta > 0$ such that

$$0 < |x-a| < \delta \Rightarrow |f(x) - l| < \frac{\varepsilon}{|K|}.$$

$$\Rightarrow |Kf(x) - Kl| < \varepsilon.$$

which proves that $\lim_{x \to a} Kf(x) = Kl = K \lim_{x \to a} f(x)$.

[Note: Here too, Kf is defined in some deleted neighbourhood of a.]

Next suppose $K = 0$. Then $Kl = 0$.

$$\therefore \lim_{x \to a} Kf(x) = \lim_{x \to a} 0 = 0.$$

$$\therefore \lim_{x \to a} Kf(x) = K \lim_{x \to a} f(x).$$

(2) Limit Of a Sequence:-

If $\{a_n\}$ is any sequence and l is a real number. If for each $\varepsilon > 0$ we can find $m \in N$ such that $n \geq m, n \in N \Rightarrow |a_n - l| < \varepsilon$, then we say that the limit of the sequence $\{a_n\}$ is l.

i.e $\lim_{n \to \infty} a_n = l$

$\lim_{n\to\infty} r^n = 0 \quad ; |r|<1.$

If $r=0$, then $r^n = 0$. $\forall n$. Hence $\lim_{n\to\infty} r^n = 0$.

Suppose $r \neq 0$ & $|r|<1$. then $|r^n - 0| < \varepsilon$

$\Leftrightarrow |r^n| < \varepsilon$

$\Leftrightarrow |r|^n < \varepsilon$

$\Leftrightarrow n \log|r| < \log \varepsilon$.

(Where we choose base of log to be >1)

Now $|r|<1 \Rightarrow \log|r| < 0$

$\therefore |r^n - 0| < \varepsilon \Leftrightarrow n > \dfrac{\log \varepsilon}{\log|r|}$

Now if $\dfrac{\log \varepsilon}{\log|r|} < 0$ (which will be the case if $\varepsilon \geq 1$)

then we can choose $m=1$. because

$n \geq 1 \Rightarrow n > \dfrac{\log \varepsilon}{\log|r|}$. So $|r^n - 0| < \varepsilon$.

and if $\dfrac{\log \varepsilon}{\log|r|} \geq 0$ (ie if $\varepsilon < 1$), then we can take

$m = \left[\dfrac{\log \varepsilon}{\log|r|} + 1\right]$. so that if $m \in N$, $n \geq m$, $n \in N$.

$n > \dfrac{\log \varepsilon}{\log|r|} \Rightarrow n \log|r| < \log \varepsilon$

$\Rightarrow \log|r^n| < \log \varepsilon$

$\Rightarrow |r^n| < \varepsilon$

$\Rightarrow |r^n - 0| < \varepsilon$

In any case, given any $\varepsilon > 0$, we can always find $m \in N$ such that $n \geq m$, $n \in N \Rightarrow |r^n - 0| < \varepsilon$.

So $\lim_{n\to\infty} r^n = 0$.

(b) (1) $\displaystyle\lim_{x\to a} \frac{\tan x - \tan a}{\tan^{-1}x - \tan^{-1}a}$

$$= \lim_{x\to a} \frac{\sin(x-a)}{\cos x \cos a} \times \frac{1-xa}{x-a} \times \frac{1}{\dfrac{\tan^{-1}\dfrac{x-a}{1+xa}}{\left(\dfrac{x-a}{1+xa}\right)}}$$

Using Working Rules,

2

$$= \lim_{x \to a} \frac{\sin(x-a)}{(x-a)} \times \lim_{x \to a} \frac{1+xa}{\cos x \cos a} \times \frac{1}{\lim_{x \to a} \frac{\tan^{-1}\frac{x-a}{1+xa}}{\frac{x-a}{1+xa}}}$$

(1)

as $x \to a$, $\frac{x-a}{1+xa} \to 0$ and $\lim_{\theta \to 0} \frac{\tan^{-1}\theta}{\theta} = 1.$

$\therefore$ From (1),

$$= 1 \times \frac{1+a^2}{\cos^2 a} \times 1$$

$$= (1+a^2)\sec^2 a$$

(2) $\lim_{h \to 0} \frac{5^{2h} - 25^{\sin h}}{h - \sin h}$

$$= \lim_{h \to 0} \frac{(25)^h - 25^{\sin h}}{h - \sin h}$$

$$= \lim_{h \to 0} 25^{\sin h} \frac{\left((25)^{h-\sin h} - 1\right)}{h - \sin h}$$

$$= \lim_{h \to 0} 25^{\sin h} \times \lim_{h \to 0} \left[\frac{(25)^{h-\sin h} - 1}{h - \sin h}\right]$$

$$= 25^0 \lim_{h \to 0} \frac{25^{h-\sin h} - 1}{h - \sin h} \quad \text{— (1)}$$

let $h - \sin h = t$ as $h \to 0$, $t \to 0$

$\therefore$ from (1)

$$= 1 \times \lim_{t \to 0} \frac{25^t - 1}{t}$$

$$= 1 \times \log_e 25$$

$$= 2 \log_e 5$$

(3) $\lim_{x \to \pi/4} (\sin 2x)^{\tan^2 2x}$

$= \lim_{x \to \pi/4} \left(1 - \cos^2 2x\right)^{\frac{\tan^2 2x}{2}}$

$= \lim_{x \to \pi/4} \left(1 - \cos^2 2x\right)^{\frac{\sec^2 2x - 1}{2}}$

$= \lim_{x \to \pi/4} \left(1 - \cos^2 2x\right)^{\frac{\sec^2 2x}{2}} \left(1 - \cos^2 2x\right)^{-1/2}$

$= \lim_{x \to \pi/4} \left[\left\{1 + (-\cos^2 2x)\right\}^{-\frac{1}{\cos^2 2x}}\right]^{-1/2} \lim_{x \to \pi/4} \left(1 - \cos^2 2x\right)^{-1/2}$

$= e^{-1/2} \times 1 = e^{-1/2}$

(c) (1) $f(x) = 2\sin x \quad ; \; x \le -\pi/2$

$= A\sin x + B \quad ; \; -\pi/2 < x < \pi/2$

$= \cos x \quad ; \; x \ge \pi/2$.

Find A & B if $f(x)$ is continuous on R.

→ Here $f(x)$ is continuous on R

∴ f is continuous at $-\pi/2, \pi/2$.

→ At $x = -\pi/2$

∴ $\lim_{x \to -\pi/2^-} f(x) = \lim_{x \to -\pi/2^-} 2\sin x = 2\sin(-\pi/2) = -2$

$\lim_{x \to -\pi/2^+} f(x) = \lim_{x \to -\pi/2^+} A\sin x + B = -A + B$

and $f(-\pi/2) = -2$.

Now, $\lim_{x \to -\pi/2^-} f(x) = f(-\pi/2) = \lim_{x \to -\pi/2^+} f(x)$

∴ $-A + B = -2 \quad \text{—— (1)}$

→ At $x = \pi/2$

∴ $\lim_{x \to \pi/2^-} f(x) = \lim_{x \to \pi/2^-} A\sin x + B = A + B$.

$\lim_{x \to \pi/2^+} f(x) = \lim_{x \to \pi/2^+} \cos x = 0$

and $f(\pi/2) = 0$

∴ $A + B = 0 \quad \text{—— (2)}$

3

by (2) ; $B = -A$

$\therefore\ -\alpha A = -\alpha$ ($\because$ from (1))

$\therefore\ A = 1\ ;\ B = -1.$

(2) (i) $\lim_{x\to 1} \dfrac{\sqrt[n]{x} + \sqrt[n]{x^2} - 2}{x-1}$

$= \lim_{x\to 1} \dfrac{(x)^{1/n} - 1 + x^{2/n} - 1}{(x-1)}$

$= \lim_{x\to 1} \dfrac{(x)^{1/n} - 1}{x-1} + \lim_{x\to 1} \dfrac{x^{2/n} - 1}{x-1}$

$= \dfrac{1}{n}(1)^{1/n - 1} + \dfrac{2}{n}(1)^{2/n - 1}$

$= \dfrac{1}{n} + \dfrac{2}{n} = \dfrac{3}{n}.$

(2) $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} 3^{r/n}$

$= \lim_{n\to\infty} \left[\dfrac{1}{n} 3^{1/n} + \frac{1}{n} 3^{2/n} + \frac{1}{n} 3^{3/n} + \dots + \frac{1}{n} 3^{n/n}\right]$

$= \lim_{n\to\infty} \left[\frac{1}{n}\left\{3^{1/n} + 3^{2/n} + 3^{3/n} + \dots + 3^{n/n}\right\}\right]$

$= \lim_{n\to\infty} \left[\dfrac{1}{n} \cdot \dfrac{3^{1/n}\left(1 - (3^{1/n})^n\right)}{1 - (3)^{1/n}}\right]$

$= \lim_{n\to\infty} \left[\dfrac{3^{1/n}.\ (1-3)}{\left\{\dfrac{1-(3)^{1/n}}{1/n}\right\}}\right]$ (as $n\to\infty \Rightarrow 1/n \to 0$)

$= \lim_{1/n\to 0} 3^{1/n} \cdot \lim_{1/n\to 0} \dfrac{2}{\dfrac{3^{1/n} - 1}{1/n}}$

$= 3^0 \times 2 \times \lim_{1/n\to 0} \dfrac{1}{\dfrac{3^{1/n} - 1}{1/n}}$

$= 1 \times 2 \times \dfrac{1}{\log_e 3} = 2 \log_3 e$

(3) $$\lim_{x\to\pi} \frac{\sqrt{2+\cos x}-1}{(\pi-x)^2}$$

$$= \lim_{x\to\pi} \frac{\sqrt{2+\cos x}-1}{(\pi-x)^2} \times \frac{\sqrt{2+\cos x}+1}{\sqrt{2+\cos x}+1}$$

$$= \lim_{x\to\pi} \frac{(2+\cos x)-1}{(\pi-x)^2\left(\sqrt{2+\cos x}+1\right)}$$

$$= \lim_{x\to\pi} \frac{1+\cos x}{(\pi-x)^2\left(\sqrt{2+\cos x}+1\right)}$$

$$= \lim_{x\to\pi} \frac{2\cos^2 x/2}{(\pi-x)^2\left(\sqrt{2+\cos x}+1\right)}$$

$$= \lim_{x\to\pi} \frac{2\sin^2\left(\frac{\pi}{2}-\frac{x}{2}\right)}{4\left(\frac{\pi}{2}-\frac{x}{2}\right)^2\left(\sqrt{2+\cos x}+1\right)}$$

$$= \lim_{x\to\pi} \frac{1}{2}\left[\frac{\sin\left(\frac{\pi}{2}-\frac{x}{2}\right)}{\left(\frac{\pi}{2}-\frac{x}{2}\right)}\right]^2 \times \lim_{x\to\pi} \frac{1}{\sqrt{2+\cos x}+1}$$

as $x\to\pi$, $\frac{\pi}{2}-\frac{x}{2}\to 0$.

$\therefore$ taking limit and using working Rules,

$$= \frac{1}{2}\times 1\times \frac{1}{\sqrt{2-1}+1} = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}.$$

(d) (1) If $f(x) = x\sin\frac{1}{x}$; $x\neq 0$ and $f(0)=0$, Prove that f is continuous but not differentiable at $x=0$.

Solution: If $x\neq 0$, $|\sin 1/x| \le 1 \Rightarrow |x\sin 1/x| \le |x|$

So, given $\varepsilon>0$, if we select $\delta=\varepsilon$, then

$0<|x-0|<\delta \Rightarrow |x|<\varepsilon \Rightarrow |x\sin 1/x| \le |x| < \varepsilon$

$\Rightarrow |x\sin 1/x - 0| < \varepsilon$

$\therefore \lim_{x\to 0} x\sin\frac{1}{x} = 0$, Also $f(0)=0$.

$\therefore f$ is continuous at $x=0$.

But $\lim_{t\to 0}\frac{f(t)-f(0)}{t-0} = \lim_{t\to 0}\frac{t\sin 1/t}{t} = \lim_{t\to 0}\sin 1/t$

which does not exist (As $\sin 1/t$ takes many values in nbhd. of 0)

(4)

$\therefore \lim_{t \to 0} \frac{f(t) - f(0)}{t - 0}$ does not exist.

$\therefore f$ is not differentiable at $x = 0$.

(2) Find the derivative at $x=0$ for $f(x) = |2x+1|$ using definition

Solution: $f(x) = |2x+1| = \begin{cases} 2x+1 & ; 2x+1 \geq 0 \text{ or } x \geq -\frac{1}{2} \\ -2x+1 & ; x < -\frac{1}{2} \end{cases}$

$$f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{(2x+1) - 1}{x} = 2$$

Que: 2 (a) (1) Prove that $f, g : (a,b) \to R$ are differentiable at x, and $g(x) \neq 0$ then $\frac{f}{g}$ is differentiable at x and

$$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{g(x) \frac{d}{dx} f(x) - f(x) \frac{d}{dx} g(x)}{(g(x))^2}.$$

→ As g is differentiable at x, it is continuous at x.

So $\lim_{t \to x} g(t) = g(x) \neq 0$. Hence there is a neighbourhood of x in (a,b) such that for all t in this nbhd. $g(t) \neq 0$

For each t in this nbhd. with $t \neq x$,

$$\lim_{t \to x} \frac{\frac{f(t)}{g(t)} - \frac{f(x)}{g(x)}}{t - x} = \lim_{t \to x} \frac{f(t) g(x) - g(t) f(x)}{g(t) g(x) (t-x)}$$

$$= \lim_{t \to x} \frac{f(t) g(x) - f(x) g(x) + f(x) g(x) - g(t) f(x)}{(t-x) g(t) g(x)}$$

$$= \frac{g(x) \lim_{t \to x} \frac{f(t) - f(x)}{t - x} - f(x) \lim_{t \to x} \frac{g(t) - g(x)}{t - x}}{g(x) \cdot \lim_{t \to x} g(t)}$$

$$= \frac{g(x) \cdot \frac{d}{dx} f(x) - f(x) \frac{d}{dx} g(x)}{[g(x)]^2}$$

[g is continuous at x.. so $\lim_{t \to x} g(t) = g(x)$]

Thus $\frac{f}{g}$ is differentiable at x

and $\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} f(x) - f(x) \frac{d}{dx} g(x)}{[g(x)]^2}$

(a) (1) Rule for derivative of a composite function.

If $f:(a,b)\to R$, $x\in(a,b)$ is differentiable at $x\in(a,b)$, if the range of f is a subset of the domain of another function g and $g:(c,d)\to R$ is differentiable at $f(x)$, then

$g\circ f:(a,b)\to R$ is differentiable at x, and

$$\frac{d}{dx}(g\circ f)(x) = g'[f(x)]\,f'(x).$$

or $(g\circ f)'(x) = g'[f(x)]f'(x)$.

(2) Mean - Value Theorem:-

If f is continuous in $[a,b]$ and differentiable in (a,b) then we can always find $c\in(a,b)$ such that $\frac{f(b)-f(a)}{b-a} = f'(c)$.

(b)

(1) $y=(\sqrt{x})^{x}+x^{\sqrt{x}}$

let $u=(\sqrt{x})^{x}$ $\quad v=x^{\sqrt{x}}$.

$\log u = x\log\sqrt{x}$

$$\therefore \frac{1}{u}\cdot\frac{du}{dx} = x\cdot\left(\frac{1}{\sqrt{x}}\right)\cdot\frac{1}{2\sqrt{x}}+\log\sqrt{x}$$

$$=\frac{x}{2x}+\log\sqrt{x}$$

$$=\frac{1}{2}+\log\sqrt{x}$$

$$\therefore \frac{du}{dx} = u\left(\frac{1}{2}+\log\sqrt{x}\right)$$

$$=(\sqrt{x})^{x}\left(\frac{1}{2}+\log\sqrt{x}\right) \quad -(1)$$

$\log v = \sqrt{x}\log x$

$$\frac{1}{v}\cdot\frac{dv}{dx} = \sqrt{x}\left(\frac{1}{x}\right)+\log x\left(\frac{1}{2\sqrt{x}}\right)$$

$$=\frac{1}{\sqrt{x}}+\frac{\log x}{2\sqrt{x}}$$

$$\frac{dv}{dx} = v\left(\frac{2+\log x}{2\sqrt{x}}\right)$$

$$=x^{\sqrt{x}}\left(\frac{2+\log x}{2\sqrt{x}}\right) \quad -(2)$$

$$\therefore \frac{dy}{dx} = \frac{du}{dx}+\frac{dv}{dx}$$

$$=(\sqrt{x})^{x}\left(\frac{1}{2}+\log\sqrt{x}\right)+x^{\sqrt{x}}\left(\frac{2+\log x}{2\sqrt{x}}\right)$$

2) $\cos^{-1}\frac{y}{b} = \log\left(\frac{x}{n}\right)^n$ (5)

$\Rightarrow \cos^{-1}\frac{y}{b} = n\log\frac{x}{n}$

$\Rightarrow \frac{y}{b} = \cos\left(n\log\frac{x}{n}\right)$

$\Rightarrow y = b\cos\left(n\log\frac{x}{n}\right)$

$\Rightarrow y_1 = -b\sin\left(n\log\frac{x}{n}\right)\times n\times\frac{n}{x}\times\frac{1}{n}$

$\Rightarrow y_1 = -\frac{bn}{x}\sin\left(n\log\frac{x}{n}\right)$

$\Rightarrow x^2y_1^2 = b^2n^2\sin^2\left(n\log\frac{x}{n}\right)$

$\Rightarrow x^2y_1^2 = b^2n^2\left(1-\cos^2\left(n\log\frac{x}{n}\right)\right)$

$\Rightarrow x^2y_1^2 = b^2n^2\left(1-\frac{y^2}{b^2}\right)$

$\Rightarrow x^2y_1^2 = n^2(b^2-y^2)$

differentiate w.r.to x,

$2x^2y_1y_2 + 2xy_1^2 = -2n^2yy_1$

dividing by $2y_1$, we get

$x^2y_2 + xy_1 + n^2y = 0.$

③ $y = \cos^{-1}x + \cos^{-1}\sqrt{1-x^2}$; $|x|<1.$

$\frac{dy}{dx} = \frac{d}{dx}(\cos^{-1}x) + \frac{d}{dx}\left(\cos^{-1}\sqrt{1-x^2}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}} + \left(\frac{-1}{\sqrt{1-(\sqrt{1-x^2})^2}}\right)\times\frac{d}{dx}\sqrt{1-x^2}$

$= \frac{-1}{\sqrt{1-x^2}} + \left(\frac{-1}{\sqrt{x^2}}\right)\left(\frac{-2x}{2\sqrt{1-x^2}}\right)$

$= \frac{-1}{\sqrt{1-x^2}} + \frac{x}{\sqrt{x^2}\sqrt{1-x^2}}$

$\Rightarrow \frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}} + \frac{x}{|x|\sqrt{1-x^2}}$

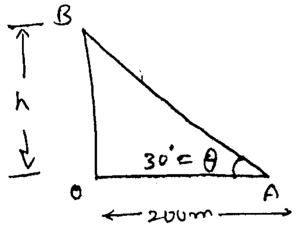
Now, $|x|<1 \Rightarrow 0<x<1$ or $-1<x<0$

If $0<x<1 \Rightarrow |x| = x \Rightarrow \frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}} + \frac{x}{x\sqrt{1-x^2}} = 0$

If $-1<x<0 \Rightarrow |x| = -x \Rightarrow \frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}} + \frac{x}{(-x)\sqrt{1-x^2}} = \frac{-2}{\sqrt{1-x^2}}$

(C) (1) from the figure,

$\tan\theta = \dfrac{h}{200}$



$\Rightarrow h = 200\tan\theta$

$\Rightarrow \dfrac{dh}{d\theta} = 200\sec^2\theta$

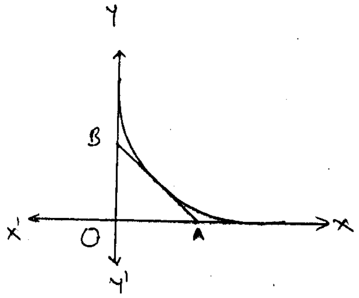
Now $\dfrac{dh}{d\theta} \approx \dfrac{\delta h}{\delta\theta}$

$\therefore \delta\theta = 12' = \dfrac{12}{60} \times \dfrac{\pi}{180}$ and $\theta = 30°$

$\therefore \delta h \approx \dfrac{12}{60} \times \dfrac{\pi}{180} \times 200\sec^2 30°$

$\therefore \delta h \approx \dfrac{1}{5} \times \dfrac{\pi}{180} \times 200\left(\dfrac{2}{\sqrt{3}}\right)^2$

$\therefore \delta h \approx \dfrac{8\pi}{27}$ m.



(2) differentiating the equation

$x^{2/3} + y^{2/3} = a^{2/3}$ w.r. to x

ie $\dfrac{2}{3}x^{-1/3} + \dfrac{2}{3}y^{-1/3}\dfrac{dy}{dx} = 0.$

$\therefore \dfrac{dy}{dx} = -\dfrac{y^{1/3}}{x^{1/3}}.$

Equation of the tangent at (x_0, y_0), of a given curve is

$y - y_0 = \dfrac{-y_0^{1/3}}{x_0^{1/3}}(x - x_0)$

$\therefore \dfrac{y - y_0}{y_0^{1/3}} = \dfrac{-(x - x_0)}{x_0^{1/3}}$

$\therefore \dfrac{y}{y_0^{1/3}} - y_0^{2/3} = \dfrac{-x}{x_0^{1/3}} + x_0^{2/3}$

$\therefore \dfrac{x}{x_0^{1/3}} + \dfrac{y}{y_0^{1/3}} = x_0^{2/3} + y_0^{2/3}$ — (1)

Also (x_0, y_0) lies on $x^{2/3} + y^{2/3} = a^{2/3}$

$\therefore x_0^{2/3} + y_0^{2/3} = a^{2/3}$ — (2)

$\therefore$ The points of intersection of the tangent with two axes are $A\left(a^{2/3}x_0^{1/3}, 0\right)$ and $B\left(0, a^{2/3}y_0^{1/3}\right)$ (6)

$$AB = \sqrt{a^{4/3}x_0^{2/3} + a^{4/3}y_0^{2/3}} = \sqrt{a^{4/3}\left(x_0^{2/3}+y_0^{2/3}\right)}$$

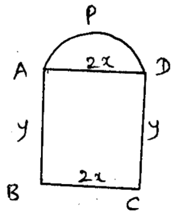
$$= \sqrt{a^{4/3} \cdot a^{2/3}} \quad (\because \text{by } (2))$$

$$= \sqrt{a^2}$$

$$= a \quad (\because a > 0)$$

$$= \text{Constant}.$$

(3)



Suppose the length of the window is $2x$ and the height is y.

$\therefore$ its perimeter is $\left(AB + BC + CD + l(\widehat{APD})\right)$

$$= 2x + 2y + \pi x$$

(length of semi circle $= \pi \cdot \frac{2x}{2} = \pi x$)

$\therefore (\pi+2)x + 2y = K$, where $K =$ constant.

For a max. amt. of light to pass through the window, its area should be max. So we wish to maximise

$$f(x) = 2xy + \frac{1}{2}\pi x^2$$

$$\therefore f(x) = \frac{1}{2}\pi x^2 + x\left(K - \pi x - 2x\right)$$

$$\therefore f'(x) = 0 \Rightarrow \pi x + K - 2\pi x - 4x = 0$$

$$\therefore K = 4x + \pi x$$

$$\therefore x = \frac{K}{\pi+4} \qquad \text{Here } f''(x) = -\pi - 4$$

Now $f''(x) < 0$

$\therefore$ length of the window $= 2x = \dfrac{2K}{\pi+4}$

and height of window $= y = \dfrac{K - \pi x - 2x}{2}$

$$= \frac{4x + \pi x - \pi x - 2x}{2}$$

$$= x$$

$\therefore$ For max. passage of light, length should be twice the height

(4) let $x = \frac{\log(1+x)}{x}$

$$\Rightarrow f'(x) = \frac{x \times \frac{1}{1+x} - \log(1+x)}{x^2}$$

$$= \frac{x - (1+x)\log(1+x)}{x^2(1+x)} \quad \text{——— (1)}$$

let $g(x) = x - (1+x)\log(1+x)$

$$\therefore g'(x) = 1 - \left[(1+x) \times \frac{1}{(1+x)} + \log(1+x)\right]$$

$$\therefore g'(x) = 1 - [1 + \log(1+x)] = -\log(1+x)$$

$g'(x) < 0$ (as $x > 0$)

$\Rightarrow g(x)$ is a decreasing function.

$\Rightarrow x > 0 \Rightarrow g(x) < g(0)$

$$\Rightarrow x - (1+x)\log(1+x) < 0 \quad \text{——— (2)}$$

from (1) & (2) $f'(x) < 0$

$\therefore f$ is a decreasing function

(d) (1) $f(x) = x + \sin 2x, \quad x \in [0, \pi]$

$\therefore f'(x) = 1 + 2\cos 2x$

$f'(x) = 0 \Rightarrow \cos 2x = -\frac{1}{2}$

$\Rightarrow 2x = 2k\pi \pm \cos^{-1}(-\frac{1}{2})$

$\Rightarrow 2x = 2k\pi \pm \frac{2\pi}{3}$; $k \in Z$

$\Rightarrow x = k\pi \pm \frac{\pi}{3}$; $k \in Z$

$\Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}$ as $x \in [0, \pi]$

Cancelled

Now $f''(x) = -4\sin 2x$

$f''(\frac{\pi}{3}) = -4\sin\frac{2\pi}{3} = -\frac{4\sqrt{3}}{2} < 0$

(d) (1) Find the local extreme values of $f(x) = x+\frac{4}{x}$; $x \in R-\{0\}$. (7)

Here $f(x) = x+\frac{4}{x}$

$\therefore f'(x) = 1-\frac{4}{x^2} = 0$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

Now $f''(x) = +\frac{8}{x^3}$ so $f''(2) = 1 > 0$

Hence $f(2)$ is a local minimum

$\therefore f(2) = 4$ is a local minimum

Now $f''(-2) = -1 < 0$ $\therefore$ $f(-2)$ is a local Max.

$f(-2) = -4$ is a local Max.

(2) $f(x) = x^3-12x$; $x \in [0,2\sqrt{3}]$

f is continuous on $[0,2\sqrt{3}]$ and differentiable on $(0,2\sqrt{3})$ as f is a polynomial function.

Now $f(0) = 0 = f(2\sqrt{3})$.

$\therefore$ Rolle's theorem is applicable

Hence $c \in (0,2\sqrt{3})$ such that $f'(c) = 0$.

$\therefore$ $3c^2-12c = 0$ $\Rightarrow c = \pm 2$

but $-2 \notin (0,2\sqrt{3})$ $\Rightarrow c = 2$

(3) $4x^2+9y^2 = 45$ — (1) $x^2-4y^2 = 5$ — (2)

By solving (1) & (2), the points of intersection are $(3,1)$, $(3,-1)$, $(-3,1)$ and $(-2,1)$

Now differentiate (1) w.r.to x then

$8x + 18y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{8x}{18y} = \frac{-4x}{9y}$

differentiate (2) w.r.to x;

$2x - 8y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{x}{4y}$

at $(3,1)$, $\left(\frac{dy}{dx}\right)_{(3,1)} = \frac{-4}{9} \times \frac{3}{1} = -4/3 = m_1$ and

$\left(\frac{dy}{dx}\right)_{(3,1)} = \frac{3}{4} = m_2$

$m_1 m_2 = -1$

$\therefore$ Curve intersects orthogonally at (3,1).

Que:3 (a) Rule of substitution of indefinite integration:

Statement: $g : [\alpha, \beta] \to R$ is continuous on $[\alpha, \beta]$ and differentiable on (α, β). $g'(t)$ is continuous on (α, β) and $g'(t) \neq 0$. $\forall\ t \in (\alpha, \beta)$.

$R_g \subset [a, b]$ and $f : [a, b] \to R$ is continuous.

Then substitution $x = g(t)$ gives,

$$\int f(x)\,dx = \int f[g(t)]\, g'(t)\, dt.$$

Now,

$$\int f(x)\,dx = F(x)$$

let $x = \frac{t-b}{a}$ ie $t = ax+b$.

Hence $x = g(t)$ is continuous and differentiable and $g'(t) = \frac{1}{a} \neq 0$. Also $g'(t)$ is continuous.

$$\frac{dx}{dt} = g'(t) = \frac{1}{a}.$$

$$\therefore \int f(ax+b)\,dx = \int f(t)\,\frac{dx}{dt}\,.\,dt = \frac{1}{a}\int f(t)\,dt$$

$$= \frac{1}{a} F(t)$$

$$= \frac{1}{a} F(ax+b).$$

OR

Rule of Integration by parts:

Statement: If (i) f, g are differentiable on interval $I \subset R$ and (ii) f', g' are continuous on I then

$$\int f(x)\, g'(x)\,dx = f(x)\,g(x) - \int f'(x)\, g(x)\,dx\ ;\quad x \in I.$$

Now we have to prove,

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c \qquad (a > 0)$$

Here $I = \int \sqrt{a^2-x^2}\, dx$ (8)

$= \int \sqrt{a^2-x^2} \cdot (1)\, dx$

$= \sqrt{a^2-x^2} \int 1\, dx - \int \left[\frac{d}{dx}\sqrt{a^2-x^2} \int 1\,dx\right] dx$

$= x\sqrt{a^2-x^2} - \int \frac{(-2x)}{2\sqrt{a^2-x^2}} \cdot x\, dx$

$= x\sqrt{a^2-x^2} - \int \frac{(a^2-x^2)-a^2}{\sqrt{a^2-x^2}}\, dx$

$= x\sqrt{a^2-x^2} - \int \sqrt{a^2-x^2}\, dx + a^2\int \frac{dx}{\sqrt{a^2-x^2}}$

$= x\sqrt{a^2-x^2} - I + a^2\sin^{-1}\frac{x}{a} + c'$

$\therefore 2I = x\sqrt{a^2-x^2} + a^2\sin^{-1}\frac{x}{a} + c'$

$\therefore I = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$ where $c = \frac{c'}{2}$

(2) f is continuous on $[0, 2a]$

$\therefore \int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

Proof: $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_a^{2a} f(x)\,dx$ as $0<a<2a$

Let $x = 2a-t \Rightarrow dx = -dt$

as $x=a$; $t=a$ & as $x=2a$; $t=0$

$\therefore \int_a^{2a} f(x)\,dx = \int_a^0 f(2a-t)(-dt) = -\int_a^0 f(2a-t)\,dt$

$= \int_0^a f(2a-t)\,dt$

$= \int_0^a f(2a-x)\,dx$

$\therefore \int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$

(b) (1) $\int \frac{x^2}{1+x^4}\,dx$

let $I = \int \frac{x^2}{1+x^4}\,dx$

$= \frac{1}{2}\int \frac{2x^2}{1+x^4}\,dx$

$= \frac{1}{2}\int \frac{(1+x^2)-(1-x^2)}{(1+x^4)}\,dx$

$= \frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx - \frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx$

$= \frac{1}{2}\int \frac{x^2(1+1/x^2)}{x^2(x^2+1/x^2)}\,dx - \frac{1}{2}\int \frac{x^2\left(\frac{1}{x^2}-1\right)}{x^2\left(\frac{1}{x^2}+x^2\right)}\,dx$

$= \frac{1}{2}\int \frac{1+1/x^2}{x^2+\frac{1}{x^2}}\,dx + \frac{1}{2}\int \frac{-\frac{1}{x^2}+1}{x^2+1/x^2}\,dx$

$= \frac{1}{2}\int \frac{1+1/x^2}{(x-1/x)^2+2}\,dx + \frac{1}{2}\int \frac{1-1/x^2}{(x+1/x)^2-2}\,dx$

$\therefore I = \frac{1}{2}I_1 + \frac{1}{2}I_2$ —— (1)

Where $I_1 = \int \frac{1+1/x^2}{(x-1/x)^2+2}\,dx$

let $x - 1/x = u$

$\Rightarrow \left(1+\frac{1}{x^2}\right)dx = du$

$\therefore I_1 = \int \frac{du}{u^2+(\sqrt{2})^2}$

$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{u}{\sqrt{2}} + c_1$

$I_2 = \int \frac{1-1/x^2}{(x+1/x)^2-2}\,dx$

let $x + 1/x = v$

$\Rightarrow \left(1-\frac{1}{x^2}\right)dx = dv$

$\therefore I_2 = \int \frac{dv}{(v)^2-(\sqrt{2})^2}$

$= \frac{1}{2\sqrt{2}}\log\left|\frac{v-\sqrt{2}}{v+\sqrt{2}}\right| + c_2$

$\therefore$ By (1)

$I = \frac{1}{2}\left[\frac{1}{\sqrt{2}}\tan^{-1}\frac{u}{\sqrt{2}} + c_1\right] + \frac{1}{2}\left[\frac{1}{2\sqrt{2}}\log\left|\frac{v-\sqrt{2}}{v+\sqrt{2}}\right| + c_2\right]$

$= \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x-1/x}{\sqrt{2}}\right) + \frac{1}{4\sqrt{2}}\log\left|\frac{x+1/x-\sqrt{2}}{x+1/x+\sqrt{2}}\right| + \frac{c_1}{2} + \frac{c_2}{2}$

$I = \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + \frac{1}{4\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right| + C \quad \left(\because \frac{c_1+c_2}{2}=C\right)$

(2) $\int x\sqrt{2ax-x^2}\,dx$ (9)

Let $I = \int x\sqrt{2ax-x^2}\,dx$

$= \int x\sqrt{a^2-(x-a)^2}\,dx$

Let $x-a = a\sin\theta \Rightarrow x = a(1+\sin\theta)$

$\Rightarrow dx = a\cos\theta\,d\theta$

$I = \int a(1+\sin\theta)\sqrt{a^2-a^2\sin^2\theta}\;a\cos\theta\,d\theta$

$= \int a(1+\sin\theta)\;a^2\cos^2\theta\,d\theta$

$= a^3\int(1+\sin\theta)\cos^2\theta\,d\theta$

$= a^3\int\cos^2\theta\,d\theta + a^3\int\cos^2\theta\sin\theta\,d\theta$

$= a^3\int\left(\frac{1+\cos2\theta}{2}\right)d\theta + a^3\int\cos^2\theta\frac{d}{d\theta}(-\cos\theta)\,d\theta$

$= \frac{a^3}{2}\int d\theta + \frac{a^3}{2}\int\cos2\theta\,d\theta - \frac{a^3}{3}\cos^3\theta + c$

$= \frac{a^3}{2}\theta + \frac{a^3}{2}\frac{\sin2\theta}{2} - \frac{a^3\cos^3\theta}{3} + c$

$I = \frac{a^3\theta}{2} + \frac{a^3}{4}(2\sin\theta\cos\theta) - \frac{a^3\cos^3\theta}{3} + c$

Now $x-a = a\sin\theta \Rightarrow \theta = \sin^{-1}\frac{(x-a)}{a}$

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{x-a}{a}\right)^2} = \frac{\sqrt{a^2-(x-a)^2}}{a}$

$\therefore \cos\theta = \frac{\sqrt{2ax-x^2}}{a}$

$\therefore I = \frac{a^3}{2}\sin^{-1}\frac{(x-a)}{a} + \frac{a^3}{2}\frac{(x-a)}{a}\cdot\frac{\sqrt{2ax-x^2}}{a} - \frac{a^3}{3}\frac{(2ax-x^2)^{3/2}}{a^3} + c$

$I = \frac{a^3}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + \frac{a}{2}(x-a)\sqrt{2ax-x^2} - \frac{(2ax-x^2)^{3/2}}{3} + c$

(3) $I = \int \sqrt{1+\cos 4x}\, dx \qquad (\pi/4 < x < \pi/2)$

$= \int \sqrt{2\cos^2 2x}\, dx$

Now $\pi/4 < x < \pi/2 \Rightarrow \pi/2 < 2x < \pi$

$\Rightarrow \cos 2x < 0$

$\therefore I = -\sqrt{2}\int \cos 2x\, dx$

$= -\frac{\sqrt{2}\sin 2x}{2} + c$

$\therefore I = \frac{-\sin 2x}{\sqrt{2}} + c$

(c) $\int_a^b \sin x\, dx$ (as limit of a sum).

Here $f(x) = \sin x$ is continuous in $[a,b]$

divide $[a,b]$ into subinterval of equal length,

each subinterval is of length $h = \frac{b-a}{n}$

as $n \to \infty$, $h \to 0$.

$\therefore I = \int_a^b \sin x\, dx = \lim_{h\to 0} h \sum_{i=1}^{n} f(a+ih)$

$= \lim_{h\to 0} h \sum_{i=1}^{n} \sin(a+ih)$

$= \lim_{h\to 0} h\, S_n$

where $S_n = \sum_{i=1}^{n} \sin(a+ih)$

$= \sin(a+h) + \sin(a+2h) + \dots + \sin(a+nh)$

Multiply by $2\sin h/2$ on both sides, we get

$2\sin\frac{h}{2} S_n = 2\sin\frac{h}{2}\sin(a+h) + 2\sin\frac{h}{2}\sin(a+2h) + \dots$
$+ 2\sin\frac{h}{2}\sin(a+nh)$

using $2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$

$2\sin\frac{h}{2} S_n = \left[\cos\left(a+\frac{h}{2}\right) - \cos\left(a+\frac{3h}{2}\right)\right] + \left[\cos\left(a+\frac{3h}{2}\right) - \cos\left(a+\frac{5h}{2}\right)\right]$
$+ \dots + \left[\cos\left(a+nh-\frac{h}{2}\right) - \cos\left(a+nh+\frac{h}{2}\right)\right]$

$$\therefore S_n = \frac{\cos(a+h/2) - \cos(b+h/2)}{2\sin h/2} \qquad \left(\because h = \frac{b-a}{n}\right)$$ (10)

$$\therefore I = \lim_{h\to 0} \frac{h\left[\cos(a+h/2) - \cos(b+h/2)\right]}{2\sin h/2}$$

$$= \lim_{h\to 0} \frac{h/2}{\sin h/2} \cdot \lim_{h\to 0}\left[\cos(a+h/2) - \cos(b+h/2)\right]$$

$$= (1)\left[\cos a - \cos b\right]$$

$$I = \cos a - \cos b$$

(2) $\displaystyle\int_0^{\pi/4} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}, \; a>0, b>0.$

$$= \int_0^{\pi/4} \frac{\sec^2 x}{a^2 - b^2\tan^2 x}\, dx$$

Let $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

if $x = 0$ then $t = 0$ and if $x = \pi/4$ then $t = 1$

$$\therefore I = \int_0^1 \frac{dt}{a^2 - b^2 t^2}$$

$$= \int_0^1 \frac{dt}{b^2\left(\frac{a^2}{b^2} - t^2\right)}$$

$$= \frac{1}{b^2}\int_0^1 \frac{dt}{\left(\frac{a}{b}\right)^2 - (t)^2}$$

$$= \frac{1}{b^2}\cdot\left[\frac{1}{2\left(\frac{a}{b}\right)} \log\left|\frac{\frac{a}{b}+t}{-\frac{a}{b}+t}\right|\right]_0^1$$

$$= \frac{1}{b^2}\left[\frac{b}{2a}\log\left|\frac{a+tb}{tb-a}\right|\right]_0^1$$

$$= \frac{1}{2ab}\left[\log\left|\frac{a+b}{b-a}\right| - \log\left|\frac{a}{-a}\right|\right]$$

$$I = \frac{1}{2ab}\log\left|\frac{a+b}{b-a}\right|$$

(3) $\int_{-\pi}^{\pi} (x^2+x) \sin 5x \, dx$

$$\therefore I = \int_{-\pi}^{\pi} x^2 \sin 5x \, dx + \int_{-\pi}^{\pi} x \sin 5x \, dx$$

$= I_1 + I_2$ where $I_1 = \int_{-\pi}^{\pi} x^2 \sin 5x \, dx$ and $I_2 = \int_{-\pi}^{\pi} x \sin 5x \, dx$

Now $x^2 \sin 5x = f(x) \Rightarrow f(-x) = (-x)^2 \sin\{5(-x)\}$

$= x^2 \sin(-5x)$

$= -x^2 \sin 5x$

$= -f(x)$

$\therefore f$ is an odd function

$$\therefore I_1 = \int_{-\pi}^{\pi} x^2 \sin 5x \, dx = 0$$

Now $x \sin 5x = g(x) \Rightarrow g(-x) = (-x) \sin\{5(-x)\}$

$= x \sin 5x = g(x)$

$\therefore g$ is an even function

$$\therefore \int_{-\pi}^{\pi} x \sin 5x \, dx = 2\int_0^{\pi} x \sin 5x \, dx$$

$$= 2\left[-x \frac{\cos 5x}{5} + \int \frac{\cos 5x}{5} dx\right]_0^{\pi}$$

$$= 2\left[-x \frac{\cos 5x}{5} + \frac{\sin 5x}{25}\right]_0^{\pi}$$

$$= 2\left[-\pi \frac{\cos 5\pi}{5} + \frac{\sin 5\pi}{25} + 0\right]$$

$$= 2\left(\frac{\pi}{5}\right)$$

$$= \frac{2\pi}{5}$$

(d) (1) $I = \int \sqrt{\frac{\sin(x-a)}{\sin(x+a)}} \, dx$

$$= \int \sqrt{\frac{\sin(x-a) \times \sin(x-a)}{\sin(x+a) \sin(x-a)}} \, dx$$

$$= \int \sqrt{\frac{\{\sin(x-a)\}^2}{\sin^2 x - \sin^2 a}} \, dx$$

$$= \int \frac{\sin(x-a)}{\sqrt{\sin^2 x - \sin^2 a}} \, dx$$

(11)

$$\therefore I = \int \frac{\sin x \cos a - \cos x \sin a}{\sqrt{\sin^2 x - \sin^2 a}} dx$$

$$= \int \frac{\sin x \cos a}{\sqrt{\sin^2 x - \sin^2 a}} dx - \int \frac{\cos x \sin a}{\sqrt{\sin^2 x - \sin^2 a}} dx$$

$$I = I_1 - I_2 \quad \text{—(1)}$$

where $I_1 = \int \frac{\sin x \cos a}{\sqrt{\sin^2 x - \sin^2 a}} dx$ and $I_2 = \int \frac{\cos x \sin a}{\sqrt{\sin^2 x - \sin^2 a}} dx$

let $\cos x = t \Rightarrow -\sin x\, dx = dt$

$$\therefore I_1 = -\int \frac{\cos a}{\sqrt{\cos^2 a - t^2}} dt = \cos a \sin^{-1} \frac{t}{\cos a}$$

$$= -\cos a \sin^{-1}\left(\frac{\cos x}{\cos a}\right)$$

for I_2, $\sin x = t \Rightarrow \cos x\, dx = dt$

$$\therefore I_2 = \sin a \int \frac{dt}{\sqrt{t^2 - \sin^2 a}} = \sin a \log\left|t + \sqrt{t^2 - \sin^2 a}\right|$$

$$\therefore I_2 = \sin a \log\left|\sin x + \sqrt{\sin^2 x - \sin^2 a}\right| + C$$

$\therefore$ by (1), $I = -\cos a \sin^{-1}\left(\frac{\cos x}{\cos a}\right) - \sin a \log\left|\sin x + \sqrt{\sin^2 x - \sin^2 a}\right| + C$

(2) $\int \cot x\, dx = \int \frac{\cos x}{\sin x} dx$

$t = \sin x$ is continuous, differentiable and non-zero on any interval not containing $k\pi = \frac{2k\pi}{2}$. $\frac{dt}{dx} = \cos x$ is continuous, differentiable and non-zero on any interval not containing $(2k+1)\frac{\pi}{2}$.

$$\int \cot x\, dx = \int \frac{dt}{t} = \log|t| + C$$

$$= \log|\sin x| + C$$

Que: 4 (a) (1)

$xy\,(x\,dy + y\,dx) = 6y^3\,dy$ and $x=2 \Rightarrow y=1$.

let $xy = z \Rightarrow x\frac{dy}{dx} + y = \frac{dz}{dx}$

$\Rightarrow x\,dy + y\,dx = dz$

$\therefore \int z\,dz = 6\int y^3\,dy$

$\Rightarrow \frac{z^2}{2} = \frac{6y^4}{4} + C$

$\Rightarrow \frac{z^2}{2} = \frac{3y^4}{2} + C$

$\Rightarrow \frac{x^2y^2}{2} = \frac{3y^4}{2} + C$

$x=2,\ y=1 \Rightarrow 2 = 3/2 + C$

$\therefore C = 1/2$

solution is $\frac{x^2y^2}{2} = \frac{3y^4}{2} + \frac{1}{2}$

$\Rightarrow x^2y^2 = 3y^4 + 1.$

(2) $\frac{dy}{dx} = \frac{2x-5y+3}{2x+4y-6}$

Solving $2x-5y+3=0$, $2x+4y-6=0$ we get the point of intersection $(h,k) = (1,1)$

Shifting origin to $(1,1)$ we get $x = x'+1$, $y = y'+1$

$\therefore dx = dx'$, $dy = dy'$

$\Rightarrow \frac{dy'}{dx'} = \frac{2x'-5y'}{2x'+4y'}$

putting $y' = Vx' \Rightarrow \frac{dy'}{dx'} = V + \frac{2x'-5y'}{2x'+4y'}$

$\Rightarrow V + x'\frac{dV}{dx'} = \frac{2-5V}{2+4V}$

$\Rightarrow x'\frac{dV}{dx'} = \frac{2-5V}{2+V(4)} - V$

$\Rightarrow x'\frac{dV}{dx'} = \frac{2-5V-2V-4V^2}{2+4V}$

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$$\Rightarrow x'\frac{dv}{dx'} = \frac{2-7v-4v^2}{2+4v}$$

$$\Rightarrow \int \frac{2+4v}{4v^2+7v-2}\,dv = -\int \frac{dx'}{x'} + \log|c|$$

$$\Rightarrow \int \frac{2+4v}{(v+2)(4v-1)}\,dv = -\log|x'| + \log|c|$$

$$= -\log|x'| + \log|c| \quad \text{--- (1)}$$

$$\therefore \frac{2+4v}{(v+2)(4v-1)} = \frac{A}{v+2} + \frac{B}{4v-1}$$

$$\Rightarrow 2+4v = A(4v-1) + B(v+2)$$

if $v=-2 \Rightarrow -6 = -9A \Rightarrow A = 2/3$

if $v = 1/4 \Rightarrow 3 = \frac{9}{4}B \Rightarrow B = 4/3.$

$$\therefore I = \int \frac{2/3}{v+2}\,dv + \int \frac{4/3}{4v-1}\,dv$$

$$= \frac{2}{3}\log|v+2| + \frac{4}{3}\frac{\log|4v-1|}{4} + \log|c'| \text{--- (2)}$$

$$\therefore I = \frac{2}{3}\log|v+2| + \frac{4}{3}\log\frac{|4v-1|}{4} + \log|c'| = -\log|x'| + \log|c|$$

$$\therefore \log(v+2)^2 + \log(4v-1) + \log|x'|^3 = \log|c|$$

$$\therefore \log\left((v+2)^2(4v-1)x'^3\right) = \log|c|$$

$$\therefore \log\left(\frac{y'}{x'}+2\right)^2\left(4\frac{y'}{x'}-1\right)x'^3 = \log|c|$$

$$\therefore (y'+2x')^2(4y'-x') = c$$

$$\therefore (2x+y-3)^2(4y-x-3) = c$$

(3) $(x+y)^2\frac{dy}{dx} = 2(x+y)^2 - 3$

let $x+y=t \Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$

$$\Rightarrow t^2\left(\frac{dt}{dx}-1\right) = 2t^2 - 3$$

$$\Rightarrow \frac{dt}{dx} - t = \alpha - \frac{3}{t^2}$$

$$\Rightarrow \frac{dt}{dx} = \alpha - \frac{3}{t^2} + t$$

$$\Rightarrow \frac{dt}{dx} = \frac{\alpha t^{\alpha} - 3 + t^2}{t^2}$$

$$\Rightarrow \frac{dt}{dx} = \frac{3t^2 - 3}{t^2}$$

$$\Rightarrow \frac{t^2}{3t^2-3}\, dt = dx$$

$$\Rightarrow \frac{1}{3}\int \frac{t^2}{t^2-1}\, dt = \int dx$$

$$\Rightarrow \frac{1}{3}\int \frac{(t^2-1)+1}{(t^2-1)}\, dt = \int dx$$

$$\Rightarrow \frac{1}{3}\int dt + \frac{1}{3}\int \frac{dt}{t^2-1} = \int dx$$

$$\Rightarrow \frac{1}{3} t + \frac{1}{6} \log\left|\frac{t-1}{t+1}\right| = x + c'$$

$$\Rightarrow \frac{1}{3}(x+y) + \frac{1}{6} \log\left|\frac{x+y-1}{x+y+1}\right| = x + c'$$

$$\Rightarrow \log\left|\frac{x+y-1}{x+y+1}\right| = 4x - 2y + C \quad (\because 6c' = C)$$

(4) let initial quantity of ice be M_0.

at $t=0$, quantity be M_0

Let M be Mass at any time t then $\frac{dM}{dt} \propto M$

$\Rightarrow \frac{dM}{dt} = kM$ (k is constant $k \neq 0$)

$\Rightarrow \int \frac{dM}{M} = k\int dt$

$\Rightarrow M = Ce^{kt}$ ——(1)

at $t=0$, $M = M_0$, $\Rightarrow M_0 = C$

$\Rightarrow M = M_0 e^{kt}$

at $t = 30$ min, $M = \frac{M_0}{2}$ $\Rightarrow \frac{M_0}{2} = M_0 e^{30k}$

$\Rightarrow e^{30k} = \frac{1}{2}$

at $t = 90$ min. $\Rightarrow M = M_0 \left(e^{30k}\right)^3$ (13)

$= \frac{M_0}{8}$

$= \frac{1}{8}$ of original quantity will remain

b (i) $x^2 - y^2 = a^2$

Put $y = 0 \Rightarrow x = \pm a$

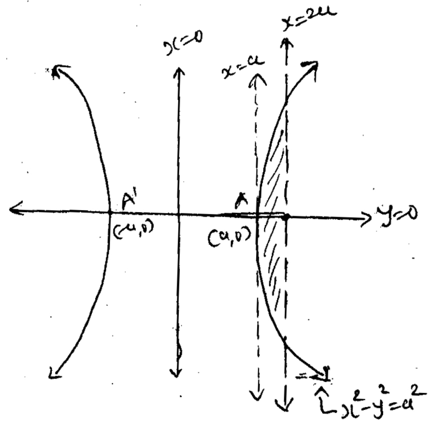
$\therefore$ The pts. of intersection are

$A(a,0)$, $A'(-a,0)$;

If $x = 2a$ then $y = \pm\sqrt{3}a$

$\therefore$ The points of intersection are

$C(2a, \sqrt{3}a)$, $D(2a, -\sqrt{3}a)$



Now $V = \pi \int_a^b y^2\, dx$

$= \pi \int_a^{2a} (x^2 - a^2)\, dx$

$= \pi \left[\frac{x^3}{3} - a^2 x\right]_a^{2a}$

$= \pi \left[\frac{8a^3}{3} - 2a^3 - \frac{a^3}{3} + a^3\right]$

$= \pi \left[\frac{7a^3}{3} - a^3\right]$

$V = \frac{4}{3}\pi a^3$

(2) Solving $x^2 + y^2 = 64$ and $y^2 = 12x$

For points of intersection, $x^2 + 12x - 64 = 0$

$\therefore (x+16)(x-4) = 0$

$\therefore x = -16$ or $x = 4$ but $x \neq -16$ ($\because y^2 < 0$) $\therefore x = 4$

$\Rightarrow y = \sqrt{12 \times 4} = \sqrt{48} = 4\sqrt{3}$

$\therefore$ The points of intersection are $A(4, 4\sqrt{3})$ and $B(4, -4\sqrt{3})$.

Now $A = \int_a^b \left[f_1(x) - f_2(x)\right] dy$

$= \int_{-4\sqrt{3}}^{4\sqrt{3}} \left[\frac{y^2}{12} - \sqrt{64 - y^2}\right] dy$

$$\therefore A = 2\int_0^{4\sqrt{3}} \left[\frac{y^2}{12} - \sqrt{64-y^2} \right] dy \qquad (\because \text{even function})$$

$$= 2\left[\frac{y^3}{36} - \left\{ \frac{y}{2}\sqrt{64-y^2} + 32\sin^{-1}\frac{y}{8} \right\} \right]_0^{4\sqrt{3}}$$

$$= 2\left[\frac{(4\sqrt{3})^3}{36} - \left\{ \frac{4\sqrt{3}}{2}\sqrt{64-48} + 32\sin^{-1}\frac{4\sqrt{3}}{8} \right\} \right]$$

$$= 2\left[\frac{64(3\sqrt{3})}{36} - \left\{ \frac{4\sqrt{3}}{2}\sqrt{16} + 32\sin^{-1}\frac{\sqrt{3}}{2} \right\} \right]$$

$$= 2\left[\frac{16}{\sqrt{3}} - \left\{ 8\sqrt{3} + 32\frac{\pi}{3} \right\} \right]$$

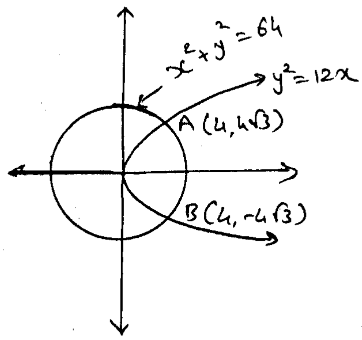
$$= 2\left[\frac{16}{\sqrt{3}} - 8\sqrt{3} - 32\frac{\pi}{3} \right]$$

$$= 2\left[\frac{-8}{\sqrt{3}} - \frac{32\pi}{3} \right]$$

$$= -\frac{16}{\sqrt{3}} - \frac{64\pi}{3}$$

$$A = -\frac{16}{3}\left[4\pi + \sqrt{3} \right]$$

$$\therefore A = \frac{16}{3}(4\pi + \sqrt{3})$$



c3) $y = 4x - x^2$, $x=1$, $x=3$ and $A = \int_a^b y\,dx$

Now $\int_1^c (4x-x^2)\,dx = \int_c^b y\,dx$

$$\therefore \int_1^c (4x-x^2)\,dx = \int_c^3 (4x-x^2)\,dx$$

$$\therefore \left[2x^2 - \frac{x^3}{3} \right]_1^c = \left[2x^2 - \frac{x^3}{3} \right]_c^3$$

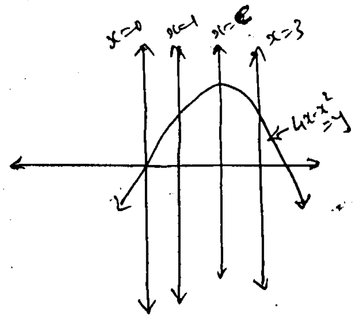
$$\therefore 2c^2 - \frac{c^3}{3} - 2 + \frac{1}{3} = 18 - \frac{27}{3} - 2c^2 + \frac{c^3}{3}$$

$$\therefore 4c^2 - \frac{2c^3}{3} - \frac{5}{3} = \frac{27}{3}$$

$$\therefore 2c^3 - 12c^2 + 32 = 0$$

$$\therefore c^3 - 6c^2 + 16 = 0$$

$$\therefore (c-2)(c^2 - 4c - 8) = 0$$



(14)

$C=2$ or $C=\frac{4\pm4\sqrt{3}}{2} = 2\pm2\sqrt{3}$

$C=2$ as $C\in(1,3)$.

$\therefore C=2$

(c) (i) $P(x) = .2$ for $x=0$

$= kx$ for $x=1,2$

$= k(6-x)$ for $x=3,4$

$= 0$ otherwise

i) $\sum P(x) = 1 \Rightarrow 0.2 + k + 2k + 3k + 2k = 1$

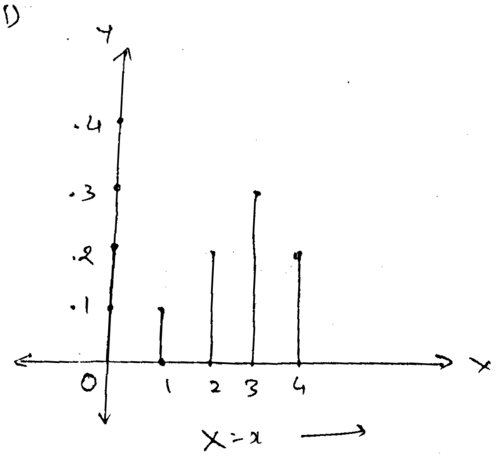
$\Rightarrow 8k = 0.8$

$\Rightarrow k = 0.1$

$\therefore$ Probability distribution is

$X(x_i)=x$	0	1	2	3	4	otherwise
$P(x)$	0.2	0.1	0.2	0.3	0.2	0

(ii)



(iii) $P(x\geq3) = P(3)+P(4)$

$= .3 + .2$

$= .5$

(iv) $V(x) = \sum x_i^2 P(x_i) - \left[\sum x_i P(x_i)\right]^2$

$= [(1)(.1)+(4)(.2)+(9)(.3)+(16)(.2)] - [1(.1)+2(.2)+3(.3)+4(.2)]^2$

$= [.1+.8+2.7+3.2] - [.1+.4+.9+.8]^2$

$= (6.8)-(2.2)^2 = 6.80 - 4.84$

$= 1.96$

(2) let x be the no. of defective products. Out of 5 selected products. Then x is a binomial random variable with parameters p & n with $n=5$.

Now the event that the no. of defective products is greater than the no. of non-defective product implies that $x \geq 3$.

$\therefore P(x \geq 3) = P(x=3) + P(x=4) + P(x=5)$

Now the probability distribution of x being binomial we have $P(x) = P(x=x) = \binom{5}{3} p^x q^{5-x}$, $x = 0,1,2,3,4,5$.

$\therefore P(x \geq 3) = \binom{5}{3} p^3 q^2 + \binom{5}{4} p^4 q + \binom{5}{5} p^5$

$= p^3(10 - 15p + 6p^2) \quad (\because q = 1-p)$

If $p = \frac{1}{2}$ then $P(x \geq 3) = \frac{1}{8}\left[10 - 15(\frac{1}{2}) + 6(\frac{1}{4})\right]$

$= \frac{1}{8}\left[10 - 7.5 + 1.5\right]$

$= \frac{1}{2}$

(3) Here random experiment of tossing balanced coin is binomial trial

$\therefore$ random variable x of getting heads is binomial random variable

Here $n = 5$, p = Probability of getting head in one toss $= \frac{1}{2}$ $\Rightarrow q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

$\therefore$ Probability distribution of x

$P(x=x) = \binom{n}{x} (\frac{1}{2})^x (\frac{1}{2})^{n-x}$; $x = 0,1,2,3,4,5$.

$P(x=0) = P(0) = \binom{5}{0}(\frac{1}{2})^0(\frac{1}{2})^5 = \frac{1}{32}$

$P(1) = \binom{5}{1}(\frac{1}{2})^5 = \frac{5}{32}$

$P(2) = \binom{5}{2}(\frac{1}{2})^5 = \frac{10}{32}$

$P(3) = \binom{5}{3}(\frac{1}{2})^5 = \frac{10}{32}$

$P(4) = \binom{5}{4}(\frac{1}{2})^5 = \frac{5}{32}$

$P(5) = \binom{5}{5}(\frac{1}{2})^5 = \frac{1}{32}$

(15)

$x(H) = x$	0	1	2	3	4	5
$P(x) = x$	$1/32$	$5/32$	$10/32$	$10/32$	$5/32$	$1/32$
$W(x)$	-16	2	4	6	8	10

$W(x)$ = Money received on getting heads

Expected gain $E(W(x))$

$= \sum W(x_i) P(x_i)$

$= -16\left(\frac{1}{32}\right) + 2\left(5/32\right) + 4\left(10/32\right) + 6\left(10/32\right) + 8\left(5/32\right) + 10\left(1/32\right)$

$= \frac{-16 + 10 + 40 + 60 + 40 + 10}{32}$

$= \frac{144}{32} = 4.50$

Expected gain Rs. 4.50

(d) (1) $I = \int 2003^{2003^{2003^x}} \cdot 2003^{2003^x} \cdot 2003^x \, dx = $ ——

let $2003^{2003^{2003^x}} = t$

$\therefore \left(2003^{2003^{2003^x}} \cdot 2003^{2003^x} \cdot 2003^x\right) dx = \frac{dt}{(\log 2003)^3}$

$\therefore I = \int \frac{dt}{(\log 2003)^3}$

$= \frac{t}{(\log 2003)^3} + c$

$I = \frac{2003^{2003^{2003^x}}}{(\log 2003)^3} + c$

(2) (i) Order of differential equation:

In the form of a polynomical the order of the higest order derivative occuring in the equation is called the order of the diff. eqn

Degree: Its power is called the degree of the diff eqn

(ii) $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ is homogeneous differential equation

Solution: let $y = vx$

$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

Substituting in differential equation,

$v + x\frac{dv}{dx} = f(v)$

$\therefore x\frac{dv}{dx} = f(v) - v$

$\therefore \int \frac{dv}{f(v) - v} = \int \frac{dx}{x}$

$\therefore F(v) = \log|x| + c$

where $F(v) = \int \frac{dv}{f(v)-v}$

~~let~~ Substituting $v = y/x$ then sol^n. of homogeneous diff. equation is

$F(y/x) = \log|x| + c$

Que: 5 (a) (a) Classical definition of Probability:

If a sample space associated with a random experiment has n possible elements (or n outcomes) if r $(0 \le r \le n)$ out of n elements are favourable to the occurence of the event A then $P(A) = \frac{r}{n}$.

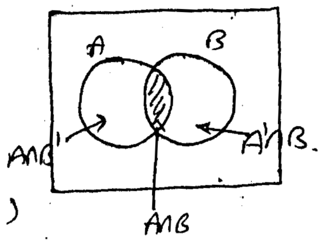
(b) Additive set function:

let $T: S \to R$ be a set function. If for all $A_1 \in S$, $A_2 \in S$ and $A_1 \cap A_2 = \phi$, $T(A_1 \cup A_2) = T(A_1) + T(A_2)$ then T defined on S is called an additive set function

OR

(15)

C(i) From the venn diagram, it is clear that events A and B-A are mutually exclusive events.



$A \cup (B\cap A') = A\cup B$.

$\therefore P(A\cup B) = P(A) + P(B\cap A')$ — (1)

Since $B\cap A' = B - (A\cap B)$ and $(A\cap B)\subset B$.

According to theorem 3, we have.

$P(B\cap A') = P(B) - P(A\cap B)$ — (2)

Hence from (1) & (2) we have

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$.

$\rightarrow P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$

C2) $$P(B_i/A) = \frac{P(B_i)\,P(A/B_i)}{P(B_1)\cdot P(A/B_1) + P(B_2)\cdot P(A/B_2)}, \quad i = 1, 2.$$

Proof: According to definition of conditional probability

$$P(B_i/A) = \frac{P(A\cap B_i)}{P(A)} \quad \text{— (1)}$$

Now according to multiplication rule of probability and theorem 6 we have,

$P(A\cap B_i) = P(B_i)\,P(A/B_i)$ — (2)

$P(A) = P(B_1)\cdot P(A/B_1) + P(B_2)\cdot P(A/B_2)$ — (3)

Using results (2) and (3) in (1) we get

$$P(B_i/A) = \frac{P(B_i)\,P(A/B_i)}{P(B_1)P(A/B_1) + P(B_2)P(A/B_2)}, \quad i = 1, 2.$$

(b) (i) U = {HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, THHT, HTTT, THTT, TTHT, TTTH, TTTT, HTHT, THTH}.

~~(ii) A - Event of getting exactly three heads~~

i) A - Event of getting at least one head

then $P(A) = \frac{15}{16}$

(ii) B = Event of getting exactly three head

$B = \{HHHT, HHTH, HTHH, THHH\}$

$P(B) = \frac{4}{16} = \frac{1}{4}$

(iii) C = Event that there are 3 or more heads.

$= \{HHHT, HHTH, HTHH, THHH, HHHH\}$

$P(C) = 5/16$

(iv) D = Event that there is no tail.

$= \{HHHH\}$

$P(D) = \frac{1}{16}$

(2) Let $\{i\}$ = Event of getting i^{th} integer on throwing a die. where $i = 1, 2, 3, 4, 5, 6$.

Now $P(\{i\}) = ki^2$

Also $\sum P(\{i\}) = 1$

$\Rightarrow k + 4k + 9k + 16k + 25k + 36k = 1.$

$\Rightarrow k = \frac{1}{91}$

Probability that integer on face of a die is even

$= P(\{2\}) + P(\{4\}) + P(\{6\})$

$= \frac{4}{91} + \frac{16}{91} + \frac{36}{91}$

$= \frac{56}{91}$

(3) No. of days in April = 30 = (4×7)+2

Two extra days can be

{(Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat), (Sat, Sun), (Sun, Mon)}.

Let r = No. of out comes in which sundays are not there in 2 extra days.

= {(Mon, Tue), (Tue, Wed), (Wed, Thu), (Thu, Fri), (Fri, Sat)} = 5

$\therefore n$ = Total outcomes = 7

$\therefore$ Reqd. Probability $= \frac{r}{n} = 5/7$

Q-5 (C) (17)

4-Red
6-White

Box-X

3-Red
7-White

Box-Y

Let B_1 = Event that the ball drawn from box X be red.

B_2 = Event that the ball drawn from box X be white.

Let A = Event that ball drawn from box Y be white.

C = Event that ball drawn from box Y be red.

Now, $P(A) = P(B_1)\cdot P(A/B_1) + P(B_2)\cdot P(A/B_2)$

$$= \frac{4}{10}\cdot\frac{7}{11} + \frac{6}{10}\cdot\frac{8}{11}$$

$$= \frac{28+48}{110} = \frac{76}{110} = \frac{38}{55}.$$

By Bayes rule

$$P(B_2/C) = \frac{P(B_2)\cdot P(C/B_2)}{P(B_1)\cdot P(C/B_1) + P(B_2)\cdot P(C/B_2)}$$

$$= \frac{\frac{6}{10}\cdot\frac{3}{11}}{\frac{4}{10}\cdot\frac{4}{11} + \frac{6}{10}\cdot\frac{3}{11}}$$

$$= \frac{18}{34} = \frac{9}{17}.$$

(2) (a) $P(A) = 0.1$, $P(B) = 0.2$

$P(A\cup B) = 0.25$, $P(A') = 0.9$

$$P(B/A') = \frac{P(B\cap A')}{P(A')} = \frac{P(B) - P(A\cap B)}{0.9}$$

$$= \frac{0.2 - 0.05}{0.9} = \frac{15}{90} = \frac{1}{6}.$$

$$P(A'/B') = \frac{P(B'\cap A')}{P(B')} = \frac{P(A\cup B)'}{1 - P(B)}$$

$$= \frac{1 - P(A\cup B)}{1 - P(B)} = \frac{1 - 0.25}{1 - 0.2}$$

$$= \dots = \frac{15}{16}$$

(b) $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{4}$ & $P(A \cap B) = \frac{1}{3}$

$A \cap B \subset B \Rightarrow P(A \cap B) \leq P(B)$

But $\frac{1}{3} \nleq \frac{1}{4}$.

$\therefore$ Allocation is not feasible.

(D) (1) 1 kilobute = 1024 bytes.

(2) $(93.625)_{10}$

	Q	R
$93 \div 2$	46	1
$46 \div 2$	23	0
$23 \div 2$	11	1
$11 \div 2$	5	1
$5 \div 2$	2	1
$2 \div 2$	1	0
$1 \div 2$	0	1

$\therefore$ $(93)_{10} = (1011101)_2$

	Product	Int. Part
$.625 \times 2 =$	1.2500	1
$.250 \times 2 =$	0.5000	0
$.500 \times 2 =$	1.0000	1

$(0.625)_{10} = (.101)_2$

$\therefore$ $(93.625)_{10} = (1011101.101)_2$

<u>Octal Form:</u> $\underline{001}\ \underline{011}\ \underline{101}.\ \underline{101}$

$(001)_2 = (1)_8$, $(011)_2 = (3)_8$, $(101)_2 = (5)_8$

$(001\,011\,101.101)_2 = (135.5)_8$

Answer: $(1011101.101)_2$ & $(135.5)_8$