QUESTION BANK
MATHS-2 (051) E

Question : 1

Q.1 (A) (1) Prove that, $\lim_{x\to 0}\frac{\sin x}{x}=1$ **(2)**

(2) Prove that, $\lim_{n\to\infty} r^n=0, |r|<|$ **(2)**

(B) Calculate any two **(4)**

(1) $\lim_{x\to 0}\frac{15^x-5^x-3^x+1}{x\log(1+x)}$

(2) $\lim_{x\to 1/2}\frac{\sin^{-1}x-\pi/6}{2x-1}$

(3) $\lim_{x\to 1}\frac{1-\sqrt{x}}{\left(\cos^{-1}x\right)^2}$

(C) (1) Find $\lim_{x\to 0}\frac{(1+mx)^x-(1+nx)^m}{x^2}$ **(2)**

(2) Calculate any two **(2)**

(i) Find the limit of the sequence $a_n=\frac{3^n+1}{3^n-1}$

(ii) $f(x)=\frac{1-\cos kx}{x^2}, \ x\neq 0$

$= 1/2 \qquad , \ x=0$

is continous at x=0, then find k.

(iii) Express 5.277777 .... as a vulgar fraction.

(D) (1) If $y=e^x(\cos x+\sin x)$ then prove that $y_2-2y_1+2y=0$ **(2)**

(2) find $\frac{dy}{dx}$ if $y=\cot^{-1}\frac{1-x^2}{2x}, \ x\neq 0, x>0$ **(1)**

Q.2 (A) (1) Define continuity on an interval and prove that $\sin x, x\in R$ is continous function. **(2)**

(2) Define limit of a sequance and prove that $a_n=\left(1+1/n\right)^n, \{a_n\} n\in N$ is bounded sequence. **(2)**

(B) Find limit of the following Any two. **(4)**

(1) $\lim_{x \to 1/3} \frac{\sin x - \sqrt{3}\cos x}{3x - \pi}$

(2) $\lim_{x \to 0} \frac{x}{\pi^{1/x} + 1}$

(3) $\lim_{x \to 1} \left(\frac{m}{1 - x^m} - \frac{n}{1 - x^n} \right) m, n \in N$

(C) (1) If $f(x) = \frac{e^{2x} - 1}{kx}, x > 0$ is continous at $x = 0$ find value of k and a **(2)**

$= |\sin x| \, x < 0$

$= a \quad x = 0$

(D) Do as directed any two. **(2)**

(1) Find $\lim_{x \to -8} x + [x]$

(2) Find $\lim_{x \to 0} (1 + 3\cos x)^{5\sec x}$

(3) Express as a vulgar fraction : 2.312 312 312 .....

(E) (1) State chain rule for finding derivative of a composite function. **(1)**

(2) If $f : (a, b) \to R$ is differentiable function at $x \in (a, b)$ then prove that it is also continues at x. **(2)**

Q.3 (A) (1) If $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then prove that

$\lim_{x \to a} \{ f(x) + g(x) \} = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$ **(2)**

(2) Prove that $\lim_{n \to \infty} r^n = 0, |r| < 1, n \in N$ **(2)**

(B) Evaluate any two limits. **(4)**

(1) $\lim_{x \to 1} (1 - x) \tan \frac{\pi x}{2}$

(2) $\lim_{n \to \infty} \sum_{r=1}^{n} \frac{1}{n} . 3^{r/n}$

(3) $\lim_{y \to 0} \left(\frac{2y + 1}{1 - 2y} \right)^{1/y}$

(C) (1) Let $f(x)=\dfrac{\log x-1}{x-e}, x\neq e$

$= k \quad , x=e$

If f is continous at $x=e$ find k. **(2)**

(2) Answer any two. **(2)**

(i) obtain the limit of the sequence 1.7, 1.77, 1.777, ...

(ii) Find the smallest $m\in N$ such that $\forall n, n\geq m, n\in N \Rightarrow |a_n-1|<0.001$

where $a_n=\dfrac{2^n+1}{2^n}$

(iii) Express $\left\{x\in R \middle/ 0\leq ||3x+1|<2\right\}$ in an interval and $N(a,\delta)$ forms.

(D) (1) Prove that $f(x)=|x-a|$ is not differentiable only at $x=a$. **(2)**

(2) Find $\dfrac{d}{dx}\left(\log_{a^n} x^n\right)$ **(1)**

Q.4 (A) (1) Prove that $\lim\limits_{\theta\to 0}\dfrac{\sin\theta}{\theta}=1$ **(2)**

(2) Prove that $\lim\limits_{n\to\infty} r^n=0; \qquad |r|<1.$ **(2)**

(B) Find the following limit (any two) **(4)**

(1) $\lim\limits_{\theta\to \pi/4}\dfrac{\sqrt{2}-\sin\theta-\cos\theta}{(4\theta-\pi)^2}$

(2) $\lim\limits_{x\to 1}\dfrac{1+\cos\pi x}{\tan^2\pi x}$

(3) $\lim\limits_{x\to 0}\dfrac{e^x-2+e^{-x}}{x\log_e(1+x)}$

(C) (1) Find $\lim\limits_{x\to 1}(1-x)\tan\dfrac{\pi}{2}x$ **(2)**

(2) Attempt any two. **(2)**

(1) $\lim\limits_{n\to\infty}\dfrac{\sum n^2}{n\sum n}$

(2) $\lim\limits_{x\to 0} x\cot 5x$

(3) If $N(a,\delta)=(1,9)$ then find δ and a.

(D) (1) Obtain $\frac{d}{dx}\left(\sqrt{\sin x}\right)$; using the definition. **(2)**

(2) If $f(x)=e^{-\log x}$, then find $f'(x)$ **(1)**

Q.5 (A) (1) If a read function f is defined every where in some deleted neighbourhood of a and $\lim\limits_{x\to a} f(x)$ exists, then prove that this limit is unique.

(2) Define limit of a sequance and prove that $\lim\limits_{n\to\infty}\frac{1}{n}=0$

(B) Solve any two. **(4)**

(1) $\lim\limits_{x\to 0}\frac{\sin x/3-1/3\sin x}{x^3}$

(2) $\lim\limits_{x\to a}\frac{e^{x^2}-e^{a^2}}{x-a}$

(3) $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{4r^2-1}$

(C) (1) Find the limit $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{n}.3^{r/n}$ **(2)**

(2) Fill in the blanks with required calculation (any two) **(2)**

(1) $\lim\limits_{n\to\infty}\frac{3^n-5^n}{3^n+5^{n+1}}=$...........$\left[0,1,-\frac{1}{5}\right]$

(2) $\{x/o<|x+2|<|;\ \ x\in R\}=$............$[(-3,-1)-\{-2\},(-1,3)-\{2\},(-3,-1)]$

(3) $\lim\limits_{n\to\infty}\frac{\sum n^3}{n\sum n^2}=$.............$\left[\frac{2}{3},\frac{3}{4},2\right]$

(D) (1) Differentiable $\cos x^2$ by definition w, r, t, x. **(2)**

(2) Show that $f(x)=|x|$ is not differentiable at 0 **(1)**

Q.6 (A) (1) P.T. $\lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta}=1$ and $0<\theta<\pi/2$ from deduce $\lim\limits_{\theta\to 0}\frac{\tan\theta}{\theta}=1$ **(2)**

(2) P.T. $\lim\limits_{n\to\infty} r^n=0|r|<|.n\in N$ **(2)**

(B) Solve any two **(4)**

(1) $\lim\limits_{x\to a}\frac{\tan x-\tan a}{\tan^{-1}x-\tan^{-1}a}$

(2) $\lim_{x \to \pi/6} \frac{2\sin x - 1}{\sqrt{3}\tan x - 1}$

(3) $\lim_{x \to 0} \frac{6^x - 3^x - 2^x + 1}{x^2}$

(C) (1) $\lim_{x \to 0} \frac{(1+mx)^n - (1+nx)^m}{x^2}$ **(2)**

(2) Fill up blanks with necessary calculations. **(2)**

(1) The range of $f : R - \left\{(2k-1)\pi/2\right\} \to R,\ f(x) = [\tan x]$ is ....

$\left(z, z - \{0\}, R^+\right)$

(2) $\lim_{x \to \infty} \left(\frac{x-1}{x+1}\right)^x = \ldots\ldots\ldots\ldots \left(e^2, 1/e^2, e, 1/e\right)$

(D) (1) Find $\frac{dy}{dx}$ if $y = \left(\sqrt{x}\right)^x + x^{\sqrt{x}}$ **(2)**

(2) Find $\frac{dy}{dx}$ if $y = \log_e\left(\sin x^2\right)$

Q.7 (A) (1) Prove that $\lim_{x \to 0} \frac{\sin x}{x} = 1$ and then prove $0 < x < \pi/2$ **(2)**

(2) Derive $\lim_{n \to \infty} r^n = 0, |r| <|. n \in N$

(B) Solve any two **(4)**

(1) $\lim_{x \to \pi/6} \frac{2\sin x - 1}{\sqrt{3}\tan x - 1}$

(2) $\lim_{x \to a} \frac{\sin^{-1} x - \sin^{-1} a}{x - a}$

(3) $\lim_{x \to 1/2} \frac{\sin^{-1} x - \pi/6}{2x - 1}$

(C) (1) Solve any two.

(i) If $a_n = \frac{1}{n}; n \in N. \forall n, n \geq m, n \in N \Rightarrow \left|\frac{1}{n} - 0\right| < \in$ then for $\in = 0.03$ Find the minimum value of m.

(ii) $\lim_{x \to 0} x\left(\sqrt[x]{5} - 1\right)$ find the limit

(iii) If $N(3,\delta)\cap N(5,\delta)=\varnothing$ then find the maximum value of δ

(D) (1) If $y=x\log\left(\frac{x}{a+bx}\right)$ then P.T. $x^3\frac{d^2y}{dx^2}=\left(x\frac{dy}{dx}-y\right)^2$

(2) If $f'(x)=f(x)$ and $f(0)=1$ then find the value of $\lim_{x\to 0}\frac{f(x)-1}{x}$

Q.8 (A) (1) Prove that $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$ **(2)**

(2) Prove that $\{an\}$; where $an=\left(1+\frac{1}{n}\right)^n; n\in N-\{1\}$ is a bounded sequnce. **(2)**

(B) Evaluate any two **(4)**

(1) $\lim_{x\to \pi/6}\frac{2\sin x-1}{\sqrt{3}\tan x-1}$

(2) $\lim_{x\to a}\frac{\tan x-\tan a}{\tan^{-1}x-\tan^{-1}a}$

(3) $\lim_{x\to 0}\frac{x\sin x}{e^x-2+e^{-x}}$

(C) (1) If $f(x)=\frac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}; x\neq 0$ then prove that $\lim_{x\to 0}f(x)$ does not exist. **(2)**

(2) Attempt any two. **(2)**

(i) If $\lim_{\theta\to 0}k\theta.\cos es\theta=\lim_{\theta\to 0}\theta\cos esk\theta$ then prove that $k=\pm 1$

(ii) Find $\lim_{n\to\infty}\frac{(n+3)!+(n+2)!}{(n+3)!-(n+2)!}$

(iii) If $\lim_{n\to\infty}\left(1+\frac{4}{n}\right)^{3n}=e^k$ then find k.

(D) (1) Find derivative of $\sin^{-1}x$ using the definition of a derivative, where $|x|<1$ **(2)**

(2) If $\log y=x$ then find y_n **(1)**

Q.9 (A) (1) P.T. Sine function is continuous on R. **(2)**

(2) P.T. $seq.\left\{\left(1+\frac{1}{n}\right)^n\right\}$ is bounded sequence and obtain its limit. **(2)**

(B) Attempt any two. **(4)**

(1) $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}}$

(2) $\lim_{x\to a}\frac{\tan x-\tan a}{\tan^{-1}x-\tan^{-1}a}$

(3) $\lim_{x\to\infty} x\sum_{i=1}^{n}\left(e^{1/x}-1\right)$

(C) (1) Find $\lim_{x\to 0}\frac{\tan^{-1}x-\sin^{-1}x}{x^3}$ **(2)**

(2) Answer the following (any 2) **(2)**

(i) Is it true that $\lim_{x\to a}\left[f(x).g(x)\right]=\lim_{x\to a}f(x).\lim_{x\to a}g(x)$? Why ?

(ii) P.T. $\lim_{x\to 0} x\sin\frac{1}{x}=0$

(iii) Find $\lim_{x\to\pi}\frac{\sin x}{\pi-x}$

(D) (i) Find $\frac{dy}{dx}$ for $y=\sqrt{t}+\cot^{-1}\sqrt{t}$ where $t=\sqrt{1+x^2}$ **(2)**

(ii) If $x^{2/3}+y^{2/3}=a^{2/3}$ then find $\frac{dy}{dx}$ **(1)**

Q.10 (A) (1) Prove that $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$ **(2)**

(2) Prove that $\lim_{n\to\infty}\frac{1}{n}=0$ **(2)**

(B) Solve any (Two) **(4)**

(1) Find $\lim_{x\to 2}\frac{(3x+3)^{1/4}-(4x+1)^{1/4}}{x^3-8}$

(2) Using definition prove that $\lim_{x\to -2}3x-1=-7$

(3) Find, $\lim_{x\to 1}\left(\frac{m}{1-x^m}-\frac{n}{1-x^n}\right), m,n\in N$

(C) (1) Find $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}(17)^{r/n}$ **(2)**

(2) Solve any 2. **(2)**

(1) Express the following as a vulgar fraction 2.123 123 123 ---------

(2) If $f(x)=\frac{\sin 2x}{3x}, x\neq 0$ continuous at $x=0$ find K.

(3) Find $\lim_{h\to 0}\frac{(x+h)^{1/4}-(x)^{1/4}}{h}$

(D) (1) For the function $x^3\sqrt{1-y^2}+y^3\sqrt{1-x^2}=a$ find $\frac{dy}{dx}$ $(|x|<|, |y|<1)$ **(2)**

(2) Find $\frac{dy}{dx}$ for $y=\log_x a$ **(1)**

Q-11 (A) (1) Prove that $\lim_{x\to 0}\frac{\sin x}{x}=1$ **(2)**

(2) Prove that $\lim_{n\to\infty} r^n=0, |r|<|$ **(2)**

(B) Calculate any two **(4)**

(1) $\lim_{x\to\infty} x\sum_{i=1}^{n}\left(e^{r/x}-1\right)$

(2) $\lim_{x\to\frac{\pi}{6}}\frac{\sqrt{3}\sin x-\cos x}{x-\frac{\pi}{6}}$

(3) $\lim_{x\to 0}\frac{15^x-5^x-3^x+1}{x\log(1+x)}$

(C) (1) Solve $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x-\sin x}}$ **(2)**

(2) **Calculate any two** **(2)**

(1) Prove that $\lim_{x\to 0}\frac{\sin^{-1}x}{x}=1$

(2) Express 3. 12 12 12 ------- as a vulgar function

(3) Examine the continuity of $f(x)=[x], x\in R$ at $x=1$

(D) (1) If $\sin y = x\sin(a+y)$ then prove $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$ **(2)**

(2) Find $\frac{d}{dx}\left[\log(\log x)\right]$

Q.12 (A) (1) If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exists then prove that

$\lim_{x\to a}\left(f(x)+g(x)\right)=\lim_{x\to a} f(x)+\lim_{x\to a} g(x)$ **(2)**

(2) Prove that $\lim_{h\to 0}\frac{a^h-1}{h}=\log e^a\ (a>0)$ and hence prove $\lim_{h\to 0}\frac{e^h-1}{h}=1$ **(2)**

(B) Attempt any two **(4)**

(1) Find $\lim_{y\to 0}\frac{(x+y)\sec(x+y)-x\sec x}{y}$

(2) Find $\lim_{x\to 0}\frac{\log(5+x)-\log(5-x)}{x}$

(3) Find $\lim_{x\to\infty} 2x\sum_{r=1}^{n}\left(e^{r/n}-1\right)$

(C) (1) Solve $\lim_{x\to\frac{\pi}{6}}\frac{2-\sqrt{3}\cos x-\sin x}{(6x-\pi)^2}$ **(2)**

(2) **Calculate any two.** **(2)**

(i) Find $\lim_{x\to\infty} x\tan\frac{1}{x}$

(ii) Prove that $\lim_{x\to 0} x\sin\frac{1}{x}=0$

(iii) Find $\lim_{n\to\infty}\sum\frac{1}{n(n+1)}$

(D) (1) If $\cos^{-1}\left(\frac{y}{b}\right)=\log\left(\frac{x}{n}\right)^x$ then prove that $x^2y_2+xy_1+n^2y=0$ **(2)**

(2) Find $\frac{d}{dx}\left(\sin x^0\right)$ **(1)**

Question : 2

Q.1 (A) (1) If $f:(a,b)\to R$ is differentiable at $x\in(a,b)$, then prove that f is continuous at x. **(2)**

(2) (i) Obtain $\frac{d}{dx}\left(\cos^{-1}x\right)=\frac{-1}{\sqrt{1-x^2}}$, where $|x|<|$ **(1)**

(ii) State the Rolle's theorem **(1)**

(B) Calculate any two **(4)**

(1) If $x=t^2-3t,\ y=t^2$ then find $\frac{d^2y}{dx^2}$

(2) If $y=x^{1/x}+(1+x)^{1/x}$ find $\frac{dy}{dx}$

(3) Find differentiation of $f(x)=\sin^2 x$ by using the definition.

(C) Calculate any two. **(4)**

(1) A particle projected vertically upwords returns to the earth in 8 seconds. If $S=ut-4.9t^2$ (S is in meters, t in sec) find the initial velocity of the particle.

(2) A circle lamina expands due to heat in such a way that its radius increases from 12.5 cm to 12.65 cm. What is the increase in area ?

(3) Verify Rolle's theorem for $f(x)=x^3-4x,\ x\in[-2,2]$

(4) Find the equation of the tangent to $9x^2+4y^2=36$ which is perpendicular to $2x-3y+1=0$

(D) (i) Prove that for $y=e^{x/c}$ the length of the subtangent is a constant at any point. **(3)**

(ii) Prove that $e^x>1+x, x\in R$

(iii) Find the approximate value of $\sin^{-1}(0.49)$

Q.2 (A) If $\frac{d}{dx}x^n=nx^{n-1}$ using this find $\frac{d}{dx}x^{1/n}$ **(1)**

(B) Do as directed any two **(4)**

(1) If $x\sqrt{1-y^2}+y\sqrt{1-x^2}=a,\ |x|<1,|y|<1$ find dy/dx

(2) If $y=e^x(\cos x+\sin x)$ prove $y_2-2y_1+2y=0$

(3) If $y=\cos^{-1}x+\cos^{-1}\sqrt{1-x^2}, |x|<1$ find dy/dx

(C) (1) If $f(a)=2, f'(a)=1, g(a)=-1, g'(a)=2$ then find $\lim_{x\to a}\frac{g(x)f(a)-f(x)g(a)}{x-a}$ **(2)**

(2) Fill up the blanks with need full calculation. **(2)**

(i) $\frac{d}{dx}\left(10^{3\log_{10}x+e^{x-1}}\right)=$.............

(ii) $\frac{d}{dx}\tan^{-1}\frac{2x}{1+8x^2}=$.............

(D) Attempt any two. **(4)**

(1) A particle projected vertically upwards returns to the earth in 8 second. If $s=ut-4.9t^2, s$ is in meters and t is in seconds, find the initial velocity of particle.

(2) If the line $y=4x-1$ touches the curve $y^2=ax^3+b$ at point (2,3) find value of a and b.

(3) Find two positive real numbers whose sum is 60 and the product of one of them with the cube of the other is maximum.

(4) By mean value theorem prove that if $x>0$ then $\frac{x}{1+x^2}<\tan^{-1}x<x$.

(E) Prove that $\frac{\tan x}{x}$ is an increasing function over $\left(0,\frac{\pi}{2}\right)$ **(2)**

Q.3 (A) (1) If is defined on (a,b) and is differentiable at $x\in(a,b)$ then prove that f is continuous at x. **(2)**

(2) (i) Prove that $\frac{d}{dx}\cos^{-1}x=\frac{-1}{\sqrt{1-x^2}}$ where $|x|<1$. **(1)**

(ii) Explain the Geometric interpretation of Rolle's Theorem. **(1)**

(B) Attempt (Any Two) **(4)**

(1) If $x=a(1-\cos\theta); y=a(\theta-\sin\theta),\quad \theta\neq k\pi,\quad k\in z$ find $\frac{d^2y}{dx^2}$ **(4)**

(2) If $y=\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ find $\frac{dy}{dx}$ if $|x|<\frac{1}{\sqrt{2}}$

(3) $xy=\sin(x+y)$ find $\frac{dy}{dx}$

(C) Answer the following (any two) **(4)**

(1) Prove that for $x>0,\ \frac{x}{1+x^2}<\tan^{-1}x<x$.

(2) Which line passing through (3,4) makes a triangle of minimum area with he axes in first quadrant.

(3) If the line $4=4x/5$ touches the curve $y^2=ax^3+b$ at (2,3) find a and b.

(4) Equation of motion of a particle projected vertically upwards is $S=pt^2+qt$ (S in meters, t in seconds). If the maximum height attained by the particle is 4.9m and if its acceleration is -9.8m/sec^2, find the height of the particle at t = 1/2.

(D) Answer the following questions. (3)

(1) Find the approximate value of $\log_{10} 99 \left(\log_{10} e=0.4343\right)$

(2) The equation of linear motion of a particle is $S=t^3-6t^2+9t+4$ find S, when a=0.

(3) Obtain the equation of the tangent $y=\sin x$ at $x=\frac{\pi}{4}$.

Q.4 (A) (1) If f is defined on (a,b) and is differentiable at $x \in (a,b)$ then P.T. f is continous at x. **(2)**

(2) Using $\frac{d}{dx}\left(\sec^{-1} x\right)=\frac{1}{|x|\sqrt{x^2-1}}, |x|>1$, deduce $\frac{d}{dx}\left(\cos ec^{-1} x\right)=$......... **(1)**

(3) State Rolle's theoram cancellation **(1)**

(B) Solve any two. (4)

(1) If $y=e^{m\tan^{-1}(x)}$, P.T. $\left(1+x^2\right)y_2+\left(2x-m\right)y_1=0$

(2) Find $\frac{dy}{dx}$ if $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right), \quad x \neq 0$

(3) Find $\frac{d^2y}{dx^2}$ if $x=a\left(\cos\theta+\log.\tan\frac{\theta}{2}\right)$ and $y=a\sin\theta$

(C) Evaluate any two (4)

(1) Sand is being collected in the form of a cone at the rate of 10 cubic meter /sec. The radius of the base is always the half of the height. Find rate at which the height increases, when the height is 5m.

(2) P.T. $4x^2+9y^2=45$ and $x^2-4y^2=5$ intersect orthogonally.

(3) Find global and local extreme values of $f(x)=x+\sin 2x, x \in [0,\pi]$

(D) Answer the following. (3)

(1) P.T. $\frac{\tan x}{x}$ is increasing function on $\left(0,\frac{\pi}{2}\right)$

(2) Apply mean value theorem, for $f(x)=\cos x, x \in [-1,0]$

(3) If the line $y=4x-5$, touches the curve $y^2=ax^3+b$ at (2,3), find a and b.

Q.2 (A) (1) If $f, g:(a,b)\to R$ are both differentiable at $x\in(a,b)$ then prove that $f.g$ is also differentiable at x and $\frac{d}{dx}\left[f(x).g(x)\right]=f(x).\frac{d}{dx}g(x)+g(x)\frac{d}{dx}f(x)$ **(2)**

(2) (a) State the rule of the derivative of an inverse function. **(1)**

(b) State the mean value theorem. **(1)**

(B) Attempt any two. **(4)**

(1) If $10^x+10^y=10^{x+y}$ then find $\frac{dy}{dx}$

(2) If $y=\sin^{-1}\left(3x-4x^3\right); \frac{1}{2}<x<1$ then find $\frac{dy}{dx}$

(3) If $x=\frac{e^t-e^{-t}}{2}$ and $y=\frac{e^t+e^{-t}}{2}$ then prove that $\frac{d^2y}{dx^2}=\frac{1}{y^3}$

(C) Answer any two. **(4)**

(1) Prove that if $\lambda_1\neq\lambda_2$ then the curves $\frac{x^2}{a^2+\lambda_1}+\frac{y^2}{b^2+\lambda_1}=1$ and $\frac{x^2}{a^2+\lambda_2}+\frac{y^2}{b^2+\lambda_2}=1$ intersect each other orthogonally.

(2) A train starts at 8 O'clock and moves eastward at the speed of 75 km/h. Another train starts from the same place at 9 O'clock and travels southward at the speed of 100 km/h. Find the rate at which they are moving away from each other at 12 O'clock.

(3) The resultant resistance R due to two resistances R_1 and R_2 is given by the formula $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$. If R_1+R_2 is constant, Prove that R is maximum when $R_1=R_2$.

(D) (1) Find the rate of increase of the area of a circle with respect to diameter. **(1)**

(2) Prove that $f(x)=\frac{\tan x}{x}$ is increasing over $\left(0,\frac{\pi}{2}\right)$ **(1)**

(3) Find an approximate value of $e^{1.002}$ where $e=2.71828$ **(1)**

Q.5 (A) (1) State the rule of derivative of Invererse function and P.T.

$\frac{d}{dx}\sec^{-1}x=\frac{1}{|x|\sqrt{x^2-1}},|x|>1$ **(2)**

(2) (i) State chain Rule **(1)**

(ii) Explain gcometric meaning of M.V. thm. **(1)**

(B) Attempt any two. **(4)**

(1) If $e^{x-y}=x^y$ then P.T. $\frac{dy}{dx}=\frac{\log\left(\frac{x}{y}\right)-\frac{1}{x}}{\log\left(\frac{x}{y}\right)-\frac{1}{y}}$

(2) If $\log y=\log\sin x-x^2$ then $y_2+4xy_1+\left(4x^2+3\right)y=0$

(3) If $x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$ and $y=a\sin\theta,\theta\neq\frac{k\pi}{2},k\in z$ then find $\frac{d^2y}{dx^2}$

(C) Attempt Any two. **(4)**

(1) Find the local extreme values of $f(x)=3x^4-10x^3+6x^2+5;\ x\in R$.

(2) The point $A(-1,1)$ and $B(2,4)$ are on the curve $y=x^2$ Find the co-ordinates targent point of the tangent parallel to $\overleftrightarrow{AB}$ of this curve.

(3) By cutting equal sequares from the four corners of a 21 x 16 tin sheet, a box to be constructed what should be the length of each square if the volume of the box is to be maximum ?

(4) If $x>0$, then P.T. $\log(1+x)>x-\frac{x^2}{2}$

(D) (1) Find the approximate value of $\sqrt[4]{80.79}$ **(1)**

(2) Find the lenght of subnormal and subtangent at the point (4,8) of the curve $y^2=x^3$ **(1)**

(3) Show that $f(x)=\tan x-x, x\in\left(0,\frac{\pi}{2}\right)$ is an increasing function. **(1)**

Q-6 (A) (1) Prove that $\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}}$ $|x|<1$ **(2)**

(2) (1) State the rule of composite derivative **(2)**

(2) State Roll's Theorem.

(B) Attempt any (Two) **(4)**

(1) If $x=at^2, y=2at\ \ t\neq 0$ then prove that $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$

(2) If $y=e^{\sqrt{x}}+e^{-\sqrt{x}}$ prove $4xy_2+2y_1=y$

(3) Using definition find derivative of $y=e^{\sin^{-1}x}$

(C) Attempt any (Two) **(4)**

(1) Find the radian measure of the angle between the tangents to $y^2 = 4ax$ and $x^2 = 4ay$ at their point of intersection other then the origen.

(2) Sand is being coelected in the form of a cone at the rate of 10 as m/sec The radius of the base is always half of the hight. Find the rate at which the height inaeases when the hight is 5m.

(3) Verify Rolles theovem for $f(x) = \sin x - \sin \alpha x, x \in [0, \pi]$

(4) Find the global and local extreme values of $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x + 1$ in [0,2]

(D) (1) Find the approximate value of $\log_{10} 999$ **(1)**

(2) Prove that $f(x) = \dfrac{x}{\sin x}$ is increasing over $\left(0, \dfrac{\pi}{2}\right)$ $(take\ f(0) = 1)$ **(1)**

(3) Find c, by applying mean value theorem to $f(x) = x^2 + x + 1, x \in [a, b]$ **(1)**

Q-7 (A) (1) Prove that if $f, g : (a,b) \to R$ both differentiable at x then fg is also differentiable at x and $\dfrac{d}{dx}[f(x).g(x)] = g(x)\dfrac{d}{dx}f(x) + f(x)\dfrac{d}{dx}g(x)$ **(2)**

(2) (1) Obtain $\dfrac{d}{dx}(\tan x) = \sec^2 x,\ x \neq (2k+1)\dfrac{\pi}{2}, k \in z$ **(1)**

(2) State the Roll's theorem **(1)**

(B) Calculate any two **(4)**

(1) Find $\dfrac{dy}{dx}$, if $y = \tan^{-1}\left(\dfrac{1-\cos x}{1+\cos x}\right)^{1/2}, \pi < x < 2\pi$

(2) Find $\dfrac{dy}{dx}$ if $y = \sin^{-1}\dfrac{2t}{1+t^2}, x = \tan^{-1}\dfrac{2t}{1-t^2}, t > 1$

(3) If $x = at^2, y = 2at, t \neq 0$ then prove that $y = \dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2 = 0$

(C) Calculate any two. **(4)**

(1) The equation of motion of a particle moving in a straight line is $s = t^3 - 5t^2 + 3t + 10$ when will it change its direction. During which intervals of time will it have the same direction.

(2) In the calculation of the area of a triangle from the formula $\Delta = \dfrac{1}{2}bc\sin A, A$

was taken as $\frac{\pi}{6}$ actually an $2\sqrt{3}\%$ error crept into this measure of A. What is the percentage error in the calculation of area. (b and c are constants)

(3) Apply mean value theorem to $f(x)=\log(1+x)$ our the interval $[0,x]$ and prove that $0<\frac{1}{\log(1+x)}-\frac{1}{x}<1\ x>0$

(4) A boy flies a kite at a height of l00m. The kite moves away from the boy at a horizontal velocity of 6.5 m/sec. Then show that the kife is 260 m away from the boy when the rate of releasing string is 6m/sec.

(D) (1) Find the approximate value of $\sqrt[4]{80.99}$ **(1)**

(2) Find the length of subnormal and subtangent at the point (4,8) of the carve $y^2=x^3$ **(1)**

(3) Show that $f(x)=\tan x-x, x\in\left(0,\frac{\pi}{2}\right)$ is an increasing function. **(1)**

Q.8 (A) (1) Prove If $f,g:(a,b)\to R$ are both differentiable at x then fg is also differentiable at x and $\frac{d}{dx}(f(x)g(x))=f(x)\frac{d}{dx}g(x)+g(x)\frac{d}{dx}f(x)$ **(2)**

(2) (i) Using division Rule of derivative prove :

$\frac{d}{dx}(\cot x)=-\operatorname{cosec}^2 x\ (x\neq k\pi, k\in z)$ **(1)**

(ii) State the second Derivative test **(1)**

(B) Calculate any two **(4)**

(1) If $\sin y=x\sin(a+y)$ prove that $\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$

(2) If $y=\left(x+\sqrt{x^2+a^2}\right)^n$ then prove that $\frac{dy}{dx}=\frac{ny}{\sqrt{x^2+a^2}}$

(3) Let $y=\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ Find $\frac{dy}{dx}$ if $\frac{1}{\sqrt{2}}<x<1$

(C) Calculate any two. **(4)**

(1) By cutting equal squares from the four corners of a 16x10 tin sheet, a box is to be constructed. What should be the length of each square, if the volume of the box is to be maximum ?

(2) Using mean value theorem, prove that if $x>0$ then $\frac{x}{1+x^2}<\tan^{-1} x$

(3) Prove that the length of tangent is constant for the curve

$$x=a\left(\cos t+\log\tan\frac{t}{2}\right)y=a\sin t$$

(4) For the curve $y=4x^3-2x^5$ find all points at which the tangent passes through the origin.

(D) (1) Find the approximate value of $\sqrt[4]{80.99}$ **(1)**

(2) Prove that $f(x)=\frac{3}{x}+7$ is decreasing for $x\in R-\{0\}$ **(1)**

(3) Apply Rolle's theorem to $f(x)=\sin x+\cos x-1\,x\in\left[0,\frac{\pi}{2}\right]$ **(1)**

Question : 3

Q.1 (A) (1) State and prove the method of substitation of integration. **(2)**

OR

Find $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$, where $0<|x|<a$

(2) If the function f is continous on $[0,2a]$ then prove

then $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(2a-x)\,dx$ **(2)**

(B) Calculate any two **(4)**

(1) $\int \frac{2x-15}{\sqrt{x^2+x+1}}\,dx$

(2) $\int \frac{1-\sin x}{1-\cos x} e^x\,dx$

(3) $\int \frac{1}{x(x^3+1)}\,dx$

(C) Find the value of any two **(4)**

(1) obtain $\int_1^2 3^x\,dx$ In as a limit of sum

(2) $\int_0^{\pi/2} \frac{dx}{3+2\sin x+\cos x}$

(3) Prove that $\int_0^{\pi} x\sin^3 x\,dx = \frac{2\pi}{3}$

(D) (1) Solve : $\int \frac{x+1}{x(1+xe^x)}\,dx$ **(2)**

(2) If $f(x)>0$ and f, f' are contiouns functions and $f'(x)\neq 0$ then find

$\int (f(x))^n f'(x)\,dx = \frac{(f(x))^{n+1}}{n+1} + c;$; where $n\neq -1$ **(1)**

Q.2 (A) (1) State and prove the rule of integration by substitution. **(2)**

OR

Prove that : $\int e^{ax}\sin(bx+c)\,dx=\frac{e^{ax}}{a^2+b^2}(a\sin bx-b\cos bx)+c$

(2) If f is continous on $[0,2a]$, prove that $\int_0^{2a} f(x)\,dx=\int_0^{a} f(x)\,dx+\int_0^{a} f(2a-x)\,dx$ **(2)**

(B) Evaluate any two. **(4)**

(1) $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$

(2) Evaluate as the limit of sum : $\int_a^b \cos x\,dx$

(3) Prove $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x+\cos x}\,dx=\frac{1}{\sqrt{2}}\log\left(\sqrt{2}+1\right)$

(C) Evaluate any two. **(4)**

(1) $\int \log\left(x+\sqrt{x^2+1}\right)dx$

(2) $\int \frac{\sin x}{\sin 3x}\,dx$

(3) $\int \frac{dx}{(x-1)^{3/2}(x-2)^{1/2}}$

(D) (1) Evaluate : $\int \sqrt{\tan x}\,dx$ **(2)**

(2) Prove : $\int e^x\{f(x)+f'(x)\}\,dx=e^x f(x)+c$ **(1)**

Q.3 (A) (1) If $\int f(x)\,dx=F(x)$ then prove that $\int f(ax+b)\,dx=\frac{1}{a}F(ax+b)\quad a\neq 0$ **(2)**

OR

Prove that

$$\int \sec x\,dx=\log\left|\left(\pi/4+x/2\right)\right|+c$$
$$=\log|\sec x+\tan x|+c,\qquad x\neq(2k+1)\pi/2,\,k\in z$$

(2) If f is continuous on $[0,a]$, then prove $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$ **(2)**

(B) Obtain the following integrals (Any Two) **(4)**

(1) $\int \frac{1}{x\left(1-x^3\right)^{1/2}} dx$

(2) $\int \frac{1}{1-\cos x+\sin x} dx$

(3) $\int \frac{2x+3}{3x+2} dx$

(C) Evaluate any two. **(4)**

(1) $\int_a^b e^x dx$ (as a limit of a sum)

(2) $\int_0^{\pi/4} \frac{1}{2+3\cos^2 x} dx$

(3) $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$

(D) (1) Obtain $\int \frac{1}{\cos 3x+\cos x} dx$ **(2)**

(2) Prove that $\int [f(x)+f'(x)] e^x dx = e^x f(x)+c$ **(1)**

Q.4 (A) (1) Prove that $\int \operatorname{cosec} x\, dx = \log\left|\tan \frac{x}{2}\right| + c$

$= \log\left|\operatorname{cosec} x - catx\right| + c$

Where, $x \neq k\pi, k \in z$

OR

If *f*, 1 are integrable functions over interval ICR then prove that

(i) $\int [f(x)+g(x)] dx = \int f(x)dx + \int g(x)dx$

(ii) $\int kf(x)dx = k\int f(x)dx; k \in R.$

(2) If function *f* is continuous over interval [0, a] then prove

$\int_0^a f(x)dx = \int_0^a f(a-x)dx$ **(2)**

(B) Solve any two. **(4)**

(1) $\int \frac{2x+3}{3x+2}dx$

(2) $\int \frac{x}{\left(x^2+9\right)\left(x^2-4\right)}dx$

(3) $\int \frac{x^2}{x^4+x^2+1}dx$

(C) Evaluate any two. **(4)**

(1) $\int_a^b \sin x\,dx$ (as a limit of a sum)

(2) $\int_{-2}^{5} |2x+1|dx$

(3) $\int_0^{\pi} \frac{x\tan x}{\sec x+\cos x}dx$

(D) (1) Find $\int \frac{1}{\cos 3x+\cos x}dx$ **(2)**

(2) Prove that $\int ten\,x\,dx=\log|\sec x|+c$ where $x\neq(2k-1)\pi/2; k\in z$ **(1)**

Q.5 (A) (1) P.T. $\int e^{ax}.\cos bx\,dx=e^{ax}\frac{\cos(bx-\theta)}{\sqrt{a^2+b^2}}+c$ **(2)**

(2) If f is an even continous function on $[-a,a]$ then P.T.

$\int_{-a}^{a} f(x)dx=2\int_0^a f(x)dx,\left(a\in R^+\right)$ **(2)**

(B) Solve any two. **(4)**

(1) $\int \frac{x^2+1}{(x+1)^2}.e^x dx$

(2) $\int \frac{x^2}{\left(x^2-9\right)\left(x^2-16\right)}dx$

(3) $\int \frac{\sqrt{25-x^2}}{x^2}dx \quad x>0$

(C) Evaluate any two. **(4)**

(1) $\int_{0}^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} . dx$

(2) $\int_{1}^{2} \frac{x^2+1}{x^4+1} . dx$

(3) $\int_{0}^{a} \frac{dx}{x + \sqrt{a^2 - x^2}}$

(D) (1) $\int \frac{1}{\cos x (1 + 2\sin x)} . dx$ **(2)**

(2) If f is contiuous and differentiable and f' is contiuous and non zero

$f(x) \neq 0 \quad \int \frac{f'(x)}{f(x)} dx = \log_e |f(x)| + c$ **(1)**

Q.6 (A) (1) State and prove the rule of integration by parts. **(2)**

OR

State and prove the rule of change of variable in integration

(2) State the fundamental principle of definite Integral and if the functions is continuous at [0, a] then prove that $\int_{0}^{a} f(x) dx = \int_{0}^{a} f(a-x) dx$

(B) Solve any two **(4)**

(1) $\int \frac{dx}{3 + 2\tan x}$

(2) $\int \frac{dx}{\sin(x-a)\sin(x-b)}$

(3) $\int \frac{\sqrt{x^2-1}}{x} dx$

(C) Solve any two **(4)**

(1) $\int_{0}^{\pi/4} \log(1 + \tan x) dx$

(2) $\int_{0}^{1} x + \tan^{-1} x \, dx$

(3) $\int_a^b \sin x\, dx$ (obtain as limit of sum)

(D) (1) $\int \sqrt{\tan x}\, dx$ **(2)**

(2) $\int \cot x\, dx = \log|\sin x| + c, x \neq \frac{n\pi}{2}; n \in z$ **(1)**

Q.7 (A) (1) State and prove the method of substitution for indefinite intergration. **(2)**

OR

Prvoe that $\int e^{ax} \cos bx\, dx = \frac{e^{ax}}{\left(a^2+b^2\right)}(a \cos bx + b \sin bx) + c$

(2) If f is continous on $[0, 2a]$ then prove that,

$\int_0^{2a} f(x)\, dx = \int_0^a f(x)\, dx + \int_0^a f(2a-x)\, dx$ **(2)**

(B) Evaluate any two **(4)**

(1) $\int \frac{1}{(x-1)^{3/2}(x-2)^{1/2}} dx$

(2) $\int \frac{x^2+1}{(x+1)^2} e^x dx$

(3) $\int \tan^{-1} \sqrt{x}\, dx$

(C) Answer any two **(4)**

(1) Evaluate : $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$

(2) Prove that, $\int_{\pi/6}^{\pi/3} \frac{e^{\sqrt{\sin x}}}{e^{\sqrt{\sin x}} + e^{\sqrt{\cos x}}} dx = \frac{\pi}{12}$

(3) Find $\int_a^b \cos x\, dx$ as the limit of a sum.

(D) (1) Evaluate : $\int \frac{1}{1+x^4} dx$ **(2)**

(2) Prove that, $\int \left[f(x)\right]^n \cdot f'(x)\,dx = \frac{\left[f(x)\right]^{n+1}}{n+1} + c;\ n \neq -1$ **(1)**

Where $f(x)>0$ and f, f' continous $f'(x)\neq 0$.

Q.8 (A) (1) State and prove rule of integration by parts. **(2)**

OR

(1) Prove $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right| + C$

(2) If f is continuous over [0, 2a] then prove. **(2)**

$$\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx$$

(B) Evaluate any two. **(4)**

(1) $\int \sqrt{\frac{x}{1-x^3}}\,dx$

(2) $\int \frac{\log x}{(1+\log x)^2}\,dx$

(3) $\int \frac{dx}{5-2\cos x-2\sqrt{3}\sin x}\,dx$

(C) Evaluate any two, **(4)**

(1) $\int_0^{\pi} \frac{x\sin x}{1+\sin x}\,dx$

(2) $\int_0^{1} \frac{\log(1+x)}{1+x^2}\,dx$

(3) $\int_{-\pi}^{\pi} \left(x^2+x\right)\sin 5x\,dx$

(D) (1) Obtain $\int_{\log_e^2}^{\log_e^5} e^x\,dx$ as the limit of the sum **(2)**

(2) Find $\int_{-2004}^{2004} \left(x^3+\sin x\right)^{2003} dx$ **(1)**

Q.9 (A) (1) State and prove the method of substitation of integration. **(2)**

OR

(1) P.T. $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}\left(a\cos bx + b\sin bx\right)+c$

$$= \frac{e^{ax}}{\sqrt{a^2+b^2}}\cos\left(bx-\theta\right)+c$$

where $\cos\theta = \frac{a}{\sqrt{a^2+b^2}}, \sin\theta = \frac{b}{\sqrt{a^2+b^2}}, a,b \neq 0$ **(2)**

(2) If f is continous on [a,b] then

P.T. $\int_a^b f\left(x\right)dx = \int_a^b f\left(a+b-x\right)dx$ **(2)**

(B) Attempt any two. **(4)**

(1) $\int\left(\frac{2+\sin 2x}{1+\cos 2x}\right)e^x\,dx$

(2) $\int\frac{\sqrt{x^2-1}}{x}dx$

(3) $\int\left(x\cos x\right)^2 dx$

(C) Attempt any two. **(4)**

(1) $\int_0^1 \frac{\log x}{\sqrt{1-x^2}}dx$

(2) $\int_0^1 \sin^{-1}\sqrt{\frac{x}{x+1}}\,dx$

(3) $\int_0^1 x\left(1+x\right)^{5/2}dx$

(D) (1) Solve : $\int\sqrt{\tan x}\,dx$ **(2)**

(2) Derive : $\int \tan x\,dx = \log|\sec x| = c, x \neq k\pi, x \neq \left(21 \leftarrow 1\right)\frac{\pi}{2}, k \in z$ **(1)**

Q.10 (A) (1) Prove that **(2)**

$$\int \sec x\,dx = \log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + c = \log|\sec x + \tan x| + c$$

$$\sec x + \tan x \neq 0 \Rightarrow 1+\sin x \neq 0 \Rightarrow x \neq (4n-1)\frac{\pi}{2}\, n \in z$$

OR

State and prove the rule of integration of by parts for indefinite integration.

(2) If f is continuous over $[a,b]$ then prove that $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ **(2)**

(B) Solve any (Two) **(4)**

(1) $\int \frac{dx}{5+4\cos x+3\sin x}$

(2) $\int \frac{dx}{(x-1)^{3/2}(x-2)^{1/2}}$

(3) $\int 2^x \cos^2 x\,dx$

(C) Solve any Two **(4)**

(1) Obtain $\int_a^b \sin x\; dx$ as the limit of a sum

(2) Prove that $\int_0^{\pi/2} \frac{x\;dx}{\sin x+\cos x} = \frac{\pi}{2\sqrt{2}}\log\left(1+\sqrt{2}\right)$

(3) Obtain $\int_0^{\pi}\left(x^2+x\right)\sin 5x\,dx$

(D) (1) Obtain $\int x^2 \cos(\log x)\,dx$ **(2)**

(2) State rule of subsituation for indefinite integration **(1)**

Q.11 (A) (1) State and prove the method of substitution of integration **(2)**

OR

(1) Prove that $\int \operatorname{cosec} x\,dx = \log\left|\tan\frac{x}{2}\right| + c = \log|\operatorname{cosec} x - \cot x| + c\ x \neq 2k\pi, k\in z$

(2) If the function f is continues and even in [-a, a] then prove that

$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$$ **(2)**

(B) Calculate any two. **(4)**

(1) $\int \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)dx \quad x>0$

(2) $\int\left(\frac{2+\sin 2x}{1+\cos 2x}\right)e^x dx$

(3) $\int \frac{\sin x}{\sin 4x}dx$

(C) Calculate any two. **(4)**

(1) $\int_1^2 \frac{x^2+1}{x^4+1}$

(2) $\int_0^{\pi/2} \frac{\cos^2 x}{\sin x+\cos x}dx$

(3) $\int_2^3 s^x\,dx$ (as a limit of sum)

(D) (1) Solve $\int \frac{x^2}{(x\sin x+\cos x)^2}dx$ **(2)**

(2) If f is continuous and differential function f' is also a continuous and $f(x)\neq 0,\ f'(x)\neq 0, x\in[a,b]$, then prove that $\int \frac{f'(x)}{f(x)}dx=\log|f(x)|+c$ **(1)**

Q.12 (A) (1) State the rule of integration by parts and prove it. **(2)**

OR

(1) State and prove the method of substitution of integration. **(2)**

(B) Calculate any two. **(4)**

(1) $\int \frac{\sqrt{\cos x}}{\sin x}dx$

(2) $\int \frac{1}{\sqrt{\sin^3 x\sin(x+2)}}dx \quad ,\propto\neq n\pi, n\in z$

(3) $\int \frac{\sin x}{\sin 4x}dx$

(C) Calculate any two. **(4)**

(1) $\int_{0}^{\pi/2}\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$

(2) $\int_{-1}^{3}|2x-1|\,dx$

(3) $\int_{0}^{\pi}\frac{x\sin x}{1+\sin x}\,dx$

(D) (1) Solve : $\int\left\{\log(\log x)+\frac{1}{(\log x)^2}\right\}dx$ **(2)**

(2) Prove : $\int\left(f(x)\right)^n f(x)\,dx=\frac{\left[f(x)\right]^{n+1}}{n+1}+c, n\neq -1$ where $f(x)>0$ and f, f' are continuous and $f'(x)\neq 0$ **(1)**

Question : 4

Q.1 (A) Calculate any two **(4)**

Solve the differential equations.

(1) $\frac{dy}{dx}=(4x+y+1)^2$

(2) $2\frac{dy}{dx}-y=e^{x/2}$

(3) $(2x+3y)dx+(y+2)dy=0$

(4) Rate of decay of uranium is proportional to its mass. If the initial mass is m_0 and half of its mass has decayed in 1200 years, how much of uranium will be left at the end of 2400 years ?

(B) Calculate any two **(4)**

(1) Find the area of the region bounded by $y=5x^2$ and $2x^2-y+9=0$

(2) Find the volume of the solid revolution generated when the region bounded by $y=x^2, y=4x-x^2$ is rotated about x - axis.

(3) Correct the following statements with necessery calculation.

(i) "The area of the regoin bounded by $y=4x$ and $y=4x^2$ is 4"

(ii) The volumn of a solid generated by upper part of the region bounded by the parabola $y^2=x$, line $x=1$ is rotated about x-axis is π unit.

(C) Calculate any two. **(4)**

(1) The mean and standard deviation of a random variable x are 10 and 5 respectively. Find $E\left(x^2\right)$, $E\left[x(x+1)\right]$ and $E\left(\frac{x-10}{5}\right)$ and $E\left(\frac{x-10}{5}\right)^2$.

(2) In a city of some western country 70 percent of the married persons take divorce. What is the probabilty that at least three among four persons will take divorce ?

(3) Three out of 10 screws in a box are defective. Four screws are selected at random from the box find the probability that out of 4 screws (i) None and (ii) One screw is detective.

(D) (1) Fill in the blanks with the necessary calculation. **(1)**

$$\int\frac{\cos x}{\sqrt{2+\sin x}}dx=.............$$

(2) (i) Define : Linear differential equation. **(1)**

(ii) Write the general form of differential equation of the first order and the first degree. **(1)**

Q.2 (A) (1) State and prove the rule of method of substiution for an inclefinite integral. **(3)**

(2) Intergrate $\int \frac{(1+x)e^x}{\cos^2\left(xe^x\right)}dx$ **(1)**

(B) Integrate any two. **(4)**

(1) $\int \frac{dx}{x\sqrt{2ax-x^2}}$

(2) $\int \frac{3x+1}{\sqrt{8-2x-x^2}}dx$

(3) $\int \frac{x^2\,dx}{1+x^4}$

(C) (1) Prove that $\int \left(f(x)+f(x)\right)e^x dx = e^x f(x)+c$ **(1)**

(2) Integrate $\int \frac{\sin x}{\sin 4x}dx$ **OR** $\int \frac{dx}{\left(1+e^x\right)\left(2+e^x\right)}$ **(2)**

(3) Find intergral of $\int \frac{x^2}{x^4-16}dx$

(D) Find integral of any two. **(2)**

(1) $\int \log x\,dx$

(2) $\int \frac{e^{2x}-1}{e^{2x}+1}dx$

(3) $\int \frac{dx}{(2x+5)\log\sqrt{2x+5}}$

Q.3 (A) Answer any two. **(4)**

(1) The vertex of a parabola is $(-a,0)$ and its latus rectum is $4a$. Prove that the differential equation is $1-\left(\frac{dy}{dx}\right)^2 = \frac{2x}{y}\cdot\frac{dy}{dx}$

(2) Solve $\frac{dy}{dx} = \sin(x+y)+\cos(x+y)$

(3) $y-$ intercept of tangent at any point (x,y) on a curve is $2xy$. Find the equation of the curve.

(4) A body having mass 60 kg. slides on the top of a table under a force $54.\sin 2t$ Newtons. Force of friction is 60 times its velocity and initially the velocity is zero. Express velocity of the body as a function of time.

(B) Answer any two. (4)

(1) Find the area of the region enclosed by $y^2 = 8x$ and $x + y = 0$

(2) Find the volume of the right circular cone having semi-vertical angle x and radius of base eqial to r.

(3) The line x = C divides the area of the region bounded by $y^2 = 4x$ and $x = 16$ in two regions having eqial areas. Find C.

(C) Answer any two. (4)

(1) Khushboo tosses a balanced die five times. It she gets 5,4,3,2 amd 1 heads, she receives Rs. 10, 8, 6, 4 and 2. respectively and loses Rs. 16 if no head is obtained. Find the expected gain of khushboo.

(2) A random variable x follows binomial distribution whose mean and variance are $10/3$ and $10/9$ respectively. Find the parameters n and p of this binomial distribution. Also find $P(x > 0)$.

(3) Four faces of a balanced die are marked with integers 1,2,3,4 and the remaining two faces are marked with 0 each. If X denotes the integer obtained on tossing the die. Find the mean and variance of the random variable x.

(D) (1) Obtain $\int \left(a^{3\log_a x} + e^{x-1}\right)dx$ **(1)**

(2) (i) Write the general form of differential equation of first order and first degree. **(1)**

(ii) Determine the order and degree of $y = x\frac{dy}{dx} + 5\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ **(1)**

Q.4 (A) Answer any two. (4)

(1) Solve $\cos(x + y)\frac{dy}{dx} = 1$

(2) Solve $x^2ydx - \left(x^3 + y^3\right)dy = 0$

(3) Solve $(2x + y)dy - (4x + 2y - 1)dy = 0$

(4) The rate of melting a piece of ice is proportional to its quantity at any moment. In 30 minutes, half of the ice melted. Prove that 1/8 th original quantity will remain after 90 minutes.

(B) Answer any two of the following. (4)

(1) Find the area of the region bounded by the curve $y = x^2 - 5x + 4$ and x axis.

(2) Find the volume of the right circular cone having semi vertical angle α and radius of the base equal to r.

(3) Obtain the area of the region bounded by, $y^2 = 8x$ and $x + y = 0$.

(C) Solve any two of the following. (4)

(1) The mean and the variance of a binomial random variable with parameter n and p are 4 and 2 respectively. Find n and p.

(2) 25% of the mangoes in a box of dozen mangoes are rotten. Three mangoes are selected at random from the box. What is the probability that one of the three mangoes is rotten.

(3) The mean and standard deviation of a random variable x are 5 and 3 respectively. Find $E\left(X^2\right), E(3X+2)^2$

(D) (1) $\int \log \sin x \cot x \, dx =$............ **(1)**

(2) Define : Differential equation.

(3) State general formula for differential equation of first order and first degree. **(1)**

Q.5 (A) Evaluate any two. (4)

(1) Find differential equation of circles having radius r and having centre on x axis.

(2) Solve the differential equation $(x+y)^2 \frac{dy}{dx} = 2(x+y)^2 - 3$

(3) Solve : $y\,dx - x\,dy + \sqrt{x^2 - y^2}\,dx = 0$

(4) The mass of a boat and sailor together is 150 kg. The sailor applies force of 70 Newton in the direction of imotion. If the repellent force is 30 times the velocity in m/s find the velocity of boat at the end of t seconds. Intially the boat is at rest.

(B) Answer the following (any 2) (4)

(1) Find the area of region bounded by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \ (a > b)$

(2) Find volume of solid generated by revolving the region bounded by $y = x^2 + 1$ and $y = 2x + 1$ about x axis.

(3) The region bounded by $y = 2x^2$ X-axis, Y-axis and $x = 5$ is rotated about Y-axis. Find volume of the solid generated.

(C) Evaluate any two. (4)

(1) Probability distribution of a discrete rendom X is defined below.

P(x) = 0.2, for x = 0

= kx, for x = 1,2

= k (6-x), for x = 3,4

(i) Find the coustant k

(ii) Draw the graph of P(x)

(iii) Find the value of $P(X \geq 3)$ and $P(0 \leq \times \leq 3)$.

(2) It has been found from an experiment that 40 percent of rats get stimulated. On administering a particular drug. If 5 rats are given this drug. What is the probability that (i) exactly three (ii) all rats get stimulated ?

(3) A player playing a game of tossing a balanced die receives Rs. 10 from his opponent if he throws 1 or 2 or 5 or 6, how much should he pay to his opponent so that the game becomes fair ?

(D) (1) Obtain $\int x^2 \log_e x\,dx$ **(1)**

(2) Define the Homogeneous differential equation and explain the method of it. **(2)**

Q.6 (A) Calculate any two. **(4)**

(1) A steamer having mass 45×10^6 starts motion with force of propeller 9,00,000 N from steady state. The force of resistance is 1,50,000 v is Newtons, where v is the velocity of the steamer in meter/sec. Express the velocity as a function of time.

(2) Solve $\left(x \sin\frac{y}{x} - y\cos\frac{y}{x}\right)dx + \cos\frac{y}{x}dy = 0$

(3) Obtain the general solution $\left(x^2 + yx^2\right)dy + \left(y^2 - xy^2\right)dy^2 = 0$

(4) $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ and find the particular solution subject to initial condition $x = 1 \,\&\, y = 1$

(B) Calculate any two **(4)**

(1) Obtain the area of the region bounded by the circle $x^2 + y^2 = 4$ and $y^2 = 3x$.

(2) The region bounded by $x^2 - y^2 = 16, y = 0$ and $x = 8$ is rotated about x axis. Find the volume of the solid generated.

(3) Correct and rewrite the following statements with necessary calculation.

(a) The area of the region bounded by $y = x^2$ and $y = x$ is $1/3$ unit.

(b) The region bounded by the parabola $y = x^2, y = 4$ and the y axis is rotated about the y axis, then the volume generated is π unit.

(C) Calculate any two. **(4)**

(1) The mean and standard deviation of a binomial random variable x are 10 and 5 respectively, then find $E\left[x(x+1)\right]$ and $E\left(\frac{x+10}{5}\right)^2$

(2) Probability distribution of a random variable is given below

$X=x$	0	1	2	3
$P(x)$	K	0	$3K$	$5K^2$

Find the acceptable value of K. Also obtain the mean of X.

(3) 400 out of a lot of 1000 screws produced in a factory are defective. From this lot 5 screws are selected at random. Find the probability that

(1) 2 screws (2) at most 1 screw is defective

(D) (1) Calculate $\int \frac{x}{\sin^2 x} dx$ **(1)**

(2) (1) Define : The differential equation **(1)**

(2) State the applications of the differential equation **(1)**

Q.7 (A) Answer any two. **(4)**

(1) Obtain the general solution of $x\frac{dy}{dx}=y+x\cos^2\left(y/x\right)$

(2) Obtain the general solution of $\frac{dy}{dx}-2y=e^{3x}\left(\tan x+\sec^2 x\right)$

(3) Obtain the general solution of $\frac{dy}{dx}+x\tan(y-x)=1$

(4) According to Newton's law of cooling the rate of cooling of a body is equal to the difference between temperature of the body and temperature of air At 20^0C air temperature the body cools from 100^0C to 60^0C in 20 minutes. When will the body temperature become 30^0C ?

(B) Answer any two. **(4)**

(1) Find the volume of the right circular cone having semi vertical angle α and radius of base equal to r.

(2) Prove that the area of the region enclosed by ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$ is πab

(3) Line $x=c$ divides the area of the region bounded by $y^2=4x$ and $x=16$ in two regions having equal area. Find c.

(C) Answer any two. **(4)**

(1) A die is constructed in such a way that the probability of getting an integer on its face is proportional to that integer A random variable X defined on the sample space U associated with the random experiment of tossing a die is as follows :

X (U) = -2, u=1,2

= 4, u=3,4

= 8, u=5,6

Obtain the probability distribution of the random variable x.

(2) Probability distribution of a random variable x is given by

$P(x)=P(X=x)=\frac{1}{m};x=1,2,3......m$. Where m is a positive integer, find the mean and variance of the random variable x.

(3) Rachana Participate in a shooting competition. The probability of her shooting a target is 0.2 What is the probability of shooting the target exactly three times out of five trials ?

(D) **(1)** Evaluate : $\int \log x\,dx$ (without using Integration by parts) **(1)**

(2) (a) Write the form of 'Linear differential Equation' and obtain its general solution. **(1)**

(b) Define : homogeneous function. **(1)**

Q.8 (A) Attempt any two. **(4)**

(1) Find the general solution of $x\frac{dy}{dx}=y+x\cos^2\left(\frac{y}{x}\right)$

(2) Find the general solution of $xy\left(xdy+ydx\right)=2y^3dy$ and also find the particular solution when $x=2\,\&\,y=1$

(3) Solve $\frac{dy}{dx}=\frac{y-x+1}{y+x+1}$

(4) The population of a town was in 1970 AD and 5000 in 1980 AD. If the rate of increase S of population is proportional to population present at that time what will be the population in 2010 AD ?

(B) Attempt any two. **(4)**

(1) The region bounded by $y=4x-x^2, x=1, x=3$ and x axis is divided in two parts with equal area by the line $x=c$ find c.

(2) The right side of the y axis of region bounded by $y=4x^2$ and $y=16$ is evoled about y axis. Find the volume of the solid generatal.

(3) Correct the following statements with calculation.

(i) The area of the region bounded by $y=\cos x, x=0, x=\pi$ and x axis b 1 unit.

(ii) The volume of a solid generated by upper part of the region bounded by $y^2=x$ line $x=1$ is rotated about x axis is π unit.

(C) Attempt any two. **(4)**

(1) Rohan tosses a balanced die 4 time. We say that a success occurs if inteqers 1 or 3 obtained on any toss. What is the probability of obtaining at most one success in 4 tosses of a die ?

(D) (1) Fill in the blank with calculation. $\int \tan^3 x \sec^2 x\, dx = ...+c$ **(1)**

(2) (i) Write the form of a honogeneous differential eqiation and state its general solution. **(1)**

(ii) Define the lines differetial equation and state its general solution. **(1)**

Q.9 (A) Solve any (Two) **(4)**

(1) Prove the differential equation of family of circle heaving centers on y axis and touching x-axis is $\left(x^2-y^2\right)\frac{dy}{dx}=2xy$

(2) Solve $\frac{dy}{dx}+y=e^x$ and if $x=0 \Rightarrow y=1$ find the particular solution.

(3) Solve $\frac{dy}{dx}=\frac{y-x+1}{y+x+5}$

(4) y - intercept of tangent at any point (x,y) on curve is $(2xy)^2$ Find the equation of the curve.

(B) Attempt any Two **(4)**

(1) Find the area of the region between the circle $x^2+y^2=4$ and $x^2+y^2=4x$

(2) the region bounded by $y=4x-x^2, x=1, x=3$ and x axis divided in two parts with equal area by $x=c$ Find c

(3) Find the volume of a sphere with radius r

(C) Solve any Two **(4)**

(1) A player playin a game of loosing a balanced die receives Rs 10 from his opponent if he throws an integer 3 or 4. If he throws 1 or 2 or 5 or 6 how much should he pay to his opponent or that the game becomes fair ?

(2) The mean and s.d of a random variable X are 10 and 5 respectively. Find $E\left(x^2\right), V(x), E\left[x(x+1)\right], E\left(\frac{x-10}{5}\right)$

(3) A balanced die is tossed five times. What is the probability the integer 1 appears at least once and exactly once.

(D) (1) Find $\int \frac{5000}{\left(x+\log x^x\right)} dx$ **(1)**

(2) Discuss the method of solving a linear differential eqn. Define Differential equation **(2)**

Q.10 (A) Calculate any two. **(4)**

(1) Solve $\frac{dy}{dx}=\sin(x+y)+\cos(x+y)$

(2) Solve $x\frac{dy}{dx}=y\left(\log y-\log x+1\right)$

(3) Solve $2\frac{dy}{dx}-y=e^{x/2}$

(4) Rate of decay of uranium is proportional to its mass at any moment. If the initial mass is m and half of its mass has decayed in 1200 years how much of uranium will be left at the end of 2400 years.

(B) Calculate any two. **(4)**

(1) Find the area of region bounded by $y=\tan x, x=\frac{\pi}{4}$ and x axis.

(2) Upper part of the region bounded by $y^2=8x$ from its wertex X to $x=2$ is rotated about x axis, find the volume of the solid generated.

(3) Correct the following statement with necessary calculation.

(i) The area of the region bounded by $y=\cos x, x=0$ $x=\pi$ and x axis is 0 unit.

(ii) The volume of a solid generated by upper part of the region bounded by the parabola $y^2=x$ line x = 1 is rotated about x axis is π unit.

(C) Calculate any two. **(4)**

(1) A random variable $x:U\to R$ where U is the sample space, associated with the experiment of a tossing of a balanced coin twice is defined as follows. for every $u\in U, x(u)=$ number of heads in u. If the elementary events of the U are equally likely then find probability distribution of x.

(2) A random variable x, assumes values -2, -1, 0, 1, 2 with the probabilities 0.1, 0.2, 0.4, 0.1, 0.2 respectfully find E(x).

(3) 3 out of 10 screws in a box are defective 4 screws are selected at random from the box. Find the probability that out of 4 screw none is defective.

(D) (1) Fill in the blanks with necessary calculation. $\int\left(\frac{x-1}{x^2}\right)e^x\,dx=...............+c$ **(1)**

(2) (i) Define : Order and degree of the differential equation. **(1)**

(ii) State the method of solation of the equation of the variable separable type and give its standard from **(1)**

Q.11 (A) Calculate any two. **(4)**

(1) Solve : $ydx-x\,dx+\sqrt{x^2-y^2}\,dx=0$ $(x>0)$

(2) Solve the differential equation : $\frac{dy}{dx}=\frac{y-x+1}{y+x+5}$

(3) y intercept of tangent at any point (x, y) an curve is $2xy^2$. Find the equ. of the curve.

(4) At any instant rate of decay of radium is proportional to its mass present. If the masses at time t_1 and t_2 are m_1 and m_2 then the time required to make the mass half of its original mass is $\dfrac{(t_2 - t_1)\log e^2}{\log e \dfrac{m_1}{m_2}}$

(B) Calculate any two. **(4)**

(1) Find the area of the region enclosed by $y^2 = 8x$ and $x + y = 0$.

(2) Find the volume of the solid obtained by revolution of portion of the ellipse on right hand semiplane of y axis about y axis.

(3) Correct following statements :

(i) Volume of solid formed when region bounded by $y = \sqrt{2}\,x$ between (0,0) and (2,2) is rotated about y axis is 8π.

(ii) Area of the region bounded by $xy = c^2, x = a, x = b \left(0 < a < b\right)$ is $c^2 \log ab$

(C) Calculate any two. **(4)**

(1) Probability of a product of a machine being defective is P. Find the probability that the number of defective products is greater than the number of non-defective products out of 5 products selected at random. If $p = \frac{1}{2}$ obtain the probability of the event.

(2) From the following probability distribution of a random variable x :

$X = x$	-1	0	1	2	3
$P(x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

find $(i)\,E(x)$ $\quad (ii)\,E\left(\dfrac{2x+3}{4}\right)^2$

(3) Three out of 10 screws in a box are defective. Four screws are selected at random from the box. Find the probability that out of 4 screws (i) none and (ii) one screws is defective.

(D) (1) Fill in the blanks with necessary calculation. $\int \dfrac{\tan x}{1 + \tan^2 x}\,dx = \ldots\ldots\ldots\ldots + c$ **(1)**

(2) (i) Define : Homogenous Differential equation. **(1)**

(ii) List the applications of differential equations. **(1)**

Q.12 (A) Calculate any two. **(4)**

(1) Solve $\left(6x^2-7y^2\right)dx-14xy=0$

(2) $\frac{dy}{dx}+y=e^{-x}$ has solution $y=\left(x+e\right)e^{-x}$ (c is arbitrany constant)

(3) According to Newton's law of cooling the rate of cooling of a body is proportional to difference between temperature of the body and the temparature of air, At 20^0C air, temperature of the body cools from 100^0C to 60^0C in 20 minutes when will the body temperature becomes 30^0C ?

(4) Solve $\frac{dy}{dx}=\frac{y-x+1}{y+x+5}$

(B) Calculate any two. **(4)**

(1) Find the area of the rigin bounded by $x=3y^2-9$ the y axis and the line $y=0$ and $y=1$.

(2) Find the volume of the solid formal when the rigin bounded by $x^2+4x=4$ and the x axis is rotated about x axis.

(3) Correct and rewrite the following statements with necessary calculation.

(i) The area of the rigion bounded by $xy=c^2$ $x=a$ and $x=b$ $\left(0<a<b\right)$ is $c^2 \log ab$

(ii) The area of the rigin bounded by $y=\cos x$ $x=0$ and $x=\pi$ is $1/2$

(C) Calculate any two. **(4)**

(1) Three faces of a balanced die is marked with integers 1,2 and 3 and the remaining sides are marked with integers 0 each. If x denotes the number & intergers obtained on tossing the die, find the mean and variance of the random variable x.

(2) 3 out at 10 screws in a box are defective 4 screws are selected at random form the box. Find the probability that out of four screws (i) none and (ii) 1 screw is defective.

(3) The probalility function of a binomial distribution is

$$P\left(x\right)=\binom{6}{x}P^x q^{6-x}, x=0,1,2......6$$

If $3P\left(x\right)=2P\left(3\right)$ then find the value of p.

(D) (1) Find $\int\frac{1}{2-\cos x}dx$ **(1)**

(2) (i) Difine : Homogeneous function. **(1)**

(ii) Explain : the general solution of a differential equation. **(1)**

Question : 5

Q.1 (A) (1) (i) For event A and B prove that $P(A\cap B')=P(A)-P(A\cap B)$ **(4)**

(ii) Define : Additive set function

OR

What is equally likely events ? In usual notations prove that the probability of the event is $P(A)=r/n$

(2) If A_1 and A_2 and A_3 are mutally indepentent events then prove that A_1 and $A_2\cup A_3$ are independent events.

(B) Calculate any two. **(4)**

(1) Two cards selected a random from pack of 52 cards. Find the probability that both the cards are (i) of black colour (ii) face cards (iii) of diamond.

(2) A box contains 5 white and some black balls. A ball is drawn at random from the box. If A denotes the event that the selected balls is black and if $P(A)=0.9$, then find the number of black balls in the box.

(3) For event A and B prove that $P(A\cap B)\geq P(A)+P(B)-1$

(C) (1) Four faces of a die are of red colour and the remaining faces are of black colour. If this die is tossed twice, find the probability that **(2)**

(i) face with red colour is obtain twice

(ii) face with black colour is obtained twice.

(2) (i) If $P(A)=0.4, P(B)=0.6$ and $P(A\cap B)=0.1$ then find $P(A\cap B')$ **(2)**

(ii) Find $P(A'/B)$ if $P(A)=0.25$, $P(B)=0.50$ and $P(A\cap B)=0.035$.

(D) (1) What we mean by the kilobyte and Megabyte ? **(1)**

(2) Convert $(216.444)_8$ into decimal number. **(2)**

Q.2 (A) Prove that

(1) For $A\in S$ $P(A')=1-P(A)$ **(1)**

(2) If $A\subset B$ then $P(B-A)=P(B)-P(A)$ $A,B\in S$ **(1)**

(3) A and B are independent events then A' an B' are also independent events. **(2)**

(B) Attempt any three. **(6)**

(1) 5 persons N, Q, R, S, T stands in a queue at the booking windows of a cinema house Find the probability of the event that

(i) Person N and Q stand adjacent to each other and

(ii) person Q is not at a first position in the queue.

(2) For two events A and B, $P(A\cup B)=0.9$, $P(A\cap B')=0.4$ and $P(A'\cap B)=0.3$

Find value of $P(A), P(B)$ $P(A'\cup B')$

(3) Box I contain 7 white and 3 black balls and box II contains 3 white and 7 black balls. Two balanced coins are tossed. If two heads are obtained box I is chosen and a ball is drown at random from it otherwise box II is chosen and a ball is drawn from it. What is the probability that the ball drawn is white ?

(4) If A, B, C are independent events and $P(A)=2\ P(B)=4P(C)=0.4$, find $P(A\cup B\cup C), P(B\cup C)$ and $P(A\cap B\cap C)$.

(C) Define : (1) Classical definition of probability.

(2) Pair wise independent events. **(2)**

(D) Attempt any three. **(3)**

(1) A,B,C,D are primary events of some sample space U. The allocation of probability given is $P(A)=0.29$, $P(B)=0.36, P(C)=0.16$ and $P(D)=0.19$. Is it possible ?

(2) For $U=\{a,b,c\}$ $P(\{a,b\})=2/3$, $P(\{a,c\})=1/3$ and $P(\{b,c\})=1/3$ distribution of probability of events possible ?

(3) If $A_1\ A_2$ are mutually exclusive events then prove that $P(A_1 / A_2')=\frac{P(A_1)}{1-P(A_2)}$

(4) It A and B are exhaustive and independent events and if $P(A)=0.2$ find $P(B)$.

Q.3 (A) (1) State and prove the rule of addition of probability. **(2)**

OR

Define : (i) Additive set function.

(ii) Classical definition of probability.

(2) State and prove Baye's rule of probability.

(B) Answer any two. **(4)**

(1) A die is constracted in such a way that the probability that integer obtained on its face when tossed, is propertional to the square of that integer. Find the probability that an interger on the face of a die is even.

(2) If for the independent events $A, B\in S, P(A)=3/4$ and $P(B)=5/8$ then prove that $1\leq 8. P(A\cap B')\leq 3$.

(3) Suppose that 3 out of 100 men and 3 out of 1000 women in a city suffer from colourblindness. A randomly selected person of the city is found to be colour-blind. It the person is a man or woman is assumed to be equally likely, what is the probability that the selected person is a woman ?

(C) (1) The probability that 60 years old man will be alive at the age of 70 years is $3/4$ and the probability that his 50 years old wife will be alive at 60 years is $2/3$. Find the probability that

(i) Only wife will be alive 10 years hence &

(ii) none of them will be alive 10 years hence. **(2)**

(2) (a) Find the probability of 53 sundays in a leap year. **(1)**

(b) A,B,C are independent events. If $P(A)=2\cdot P(B)=4\cdot P(C)=0.4$ find $P(A\cup B\cup C)$ **(1)**

(D) (1) Distingaish between bit and byte. **(1)**

(2) Convert $(25.1875)_{10}$ in octal form. **(2)**

Q.4 (A) (1) Define Axiomatic definition of probability and pair wise independent events. **(2)**

OR

If A and B are two events prove $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ and write the formula for $P(A\cup B\cup C)$

(2) If A and B are independent events, then prove that A' and B' are also independent events, also prove that A and B' are also independent. **(2)**

(B) Answer any Two of the following. **(4)**

(1) 13 cards are selected at random from a pack of 52 cards. Find the probability that 3 out of 13 cards are kings.

(2) For two events A and B, $P(A\cup B)=0.9, P(A\cap B)=0.4$ and $P(B\cap A)=0.3$ Find the value of $P(A), P(B)$ and $P(A'\cup B')$.

(3) If E and F are indepenent events and if G denotes the event that only one out of events E and F occurs then show that -

$P(G)=P(E)+P(F)-2P(E).P(F)$

(C) (1) Suppose that 3 out of 100 men and 3 out of 1000 women in a city suffer from colour blindness. A random selected person of the city is found to be colour blind. If the person is a man or a woman is assumed to be equally likely, what is the probability that the selected person is a woman.

(2) (i) From a pack of 52 cards, four cards are selected without replacement. Find the probability that they all may have different colour.

(ii) Prove $P(A'/B)=1-P(A/B)$ **(1)**

(D) (1) 1 kilo byte = ... bytes. **(1)**

(2) Convert decimal number $(39.625)_{10}$ into binary form. **(2)**

Q.5 (A) (1) If A and B are events then P.T. $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ **(2)**

(2) State and prove Bayes Rule

(B) Answer any two. **(4)**

(1) For events A and B, $P(A)=P(B)=\frac{1}{2}$ and $P(A\cup B)=\frac{2}{3}$ find the values of the following.

$(i)\ P(A\cap B)\ (ii)(A'\cap B)(iii)\ P(A'\cap B')\ (iv)\ P(A'\cup B)$

(2) A card is selected at random from a pack of 52 cards if A denotes the event that the card drawn is a card of heart, find the value of P(A)

(3) A box has 10 black and 4 white balls. If 3 balls are drawn at random from the box, what is the probability of the event that

(i) All are white ?

(ii) Atleast one is white ?

(iii) At most one is white ?

(C) Box I contains 4 red and 6 white balls and box II contains 3 red and 7 white balls. A ball is drawn at random from the box I and is transferred to box II. Now a ball is drawn at random from box II. What is the probability that the selected ball is white ? **(2)**

(D) (1) Find probability of getting 5 Mondays in month of April.

(2) Find the probability of geting 72 marks in paper in your hand. **(1)**

(E) (1) State the main parts of computer **(1)**

(2) Write the binary form of the decimal number, $(1000)_{10}$

(3) Subtract 0.4891 E-07 from 0.1645-05. **(1)**

Q.6 (A) (1) For event A and B If $A\subset B$ then **(2)**

(i) $P(B-A)=P(B)-P(A)$

(ii) $P(A)\le P(B)$

OR

P.T. $P(A\cup B\cup C)=$

$P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)$

(2) Prove that the set function $P(A/B)$ treated as a function of event A for fixed event B is aprobability function on S.

(B) Solve any two. **(4)**

(1) If P(E)=0.4, P(F) = 0.2 and P(E∪F) = 0.5 find P(E/F), P(F/E) and P(E'/F).

(2) 10 student are given find the probability two paticular persons sited to gather.

(3) A_1, A_2, A_3, A_4 are independent event then prove that A_1 and $(A_2\cup A_3)$ are also independent.

(C) (1) For event A and B P.T. $P(A\cap B)\ge P(A)+P(B)-1$ and

$P(B)=0.3, P(A\cup B)=0.7$

and $P(B/A)=0.2$ then find P(A) **(2)**

(2) (i) $P(A)=0.1,\ P(B)=0.2, P(A\cup B)=0.25$ then find $P(A/B)$ **(1)**

(ii) What is probability to you get 74 marks in this paper. **(1)**

(D) (1) Give a difference between RAM and ROM **(1)**

(2) Convert the number $(32.34)_8$ into decimal and binary system. **(2)**

Q.7 (A) (1) If A and B are events and if $A\subset B$ then prove that $P(B-A)=P(B)-P(A)$ and $P(A)\le P(B)$ **(2)**

OR

If A is some event then prove that,

$P(A')=1-P(A)$ and hence show that

$P(\varnothing)=0$

(2) State and prove 'Bayes rule' for two events. **(2)**

(B) Attempt any two. **(4)**

(1) If A and B are independent events and if C denotes the event that only one out of events A and B occurs then show that.

$P(C)=P(A)+P(B)-2\,P(A).P(B)$

(2) Suppose that 3 out of 100 men and 3 out of 1000 women in a city suffer from colour blindness. A random selected person of the city is found to be colour blind. If a person is a man or woman is assumed to be equally likely, what is the probability that the selected person is a woman ?

(3) A box contain 10 balls amon xg which 3 are black and 7 are white. Now following game is played.

'At each trail a ball is selected at random, its colour is noted and then it is replace back into the box with 2 more balls of the same colour. Find the probability of first three balls to be black.

(C) (1) For two events A and B $P(A\cup B)=0.9, P(A\cap B')=0.4$ and $P(A'\cap B)=0.3$ find the value of $P(A)$ and $P(A'\cup B')$ **(2)**

(2) (a) If A and B are mutually exclusive events then prove

that $P\left(A/B'\right)=\dfrac{P(A)}{1-P(B)}$ **(1)**

(b) If A and B exhaustive and independent events and if $P(A)=0.2$ find $P(B)$ **(1)**

(D) (1) Distinguish between bit and byte. **(1)**

(2) Convert $(32.34)_8$ into binary numbers and decimal numbers.

Q.8 (A) (1) For event A, B of $A\subset B$ then

P.T. $(i)\,P(B-A)=P(B)-P(A)$ $(ii)\,P(A)\le P(B)$ **(2)**

OR

(1) (i) P.T. $P(\varnothing)=0$

(ii) Define : Mutually exclusive and exhaustive events. **(1)**

(2) State and prove : Bayes Rule **(2)**

(B) Attempt any two. **(4)**

(1) Two faces of balanced die are marked with interger 2 and 3 and other sides are blank. If we toss blanced die five times then what the probability of getting sum of intergers exactly 12 ?

(2) If E and F are indepedent events and if G denotes the event that only one out of E and F occurs then P.T. $P(G)=P(E)+P(F)-2P(E)P(F)$

(3) There are 3 black, 6 while and 4 red balls in a box. 3 balls are drawn at random without replacement. Find the porbability that all of them are of same colour.

(C) (1) If from pack of 52 playing card. Two card are selected and keep a side. Then what is the probability of getting a card of ace from remaining cards ? **(2)**

(2) (i) If $P(A\cup B)=\frac{11}{12}$ and $P(A)=\frac{1}{3}$ then $P(B\cap A')=$............ **(1)**

(ii) A box contains 5 white and some black ball. A ball is drawn at random from the box. It A denotes the event that the selected ball is balck and if P(A)=0.9, find the number of black balls in the box. **(1)**

(D) (1) What are main components if a computers ? **(1)**

(2) Convert $(39.625)_{10}$ in binary and octal form. **(2)**

Q.9 (A) (1) Define : (i) Additive set function. **(1)**

(ii) Partition of the sample space. **(1)**

OR

(1) For events A and B prove that $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ **(2)**

(2) State and prove the rule of buyes' for two events. **(2)**

(B) Calculate any two. **(4)**

(1) 5 men and 5 women are seated at random in a row. Find the probability of the event that in a row (1) 5 women are seated together. (2) No. 2 Women are seated together.

(2) A box contains 5 white 4 black and 6 red balls. 3 balls are selected at random from it. Find the probability that there is a ball of each colour.

(3) If, A1, A2 and A3 are independent event, then prove that A1' and $A2\cap A3'$ are independent events.

(C) (1) A and B are two independent witness of some incident and the probabilities of speaking true of A and B are respectively x and y. If for same incident A and B are agree to each other then show that the probability of that incident happens is $\frac{xy}{1-x-y+2xy}$ **(2)**

(2) (i) $P(A)=0.1,\ P(B)=0.2\ P(A\cup B)=0.25$ the find $P(A/B)$ **(1)**

(ii) $P(A\cup B)=0.8,\ P(A\cap B')=0.3$ and $P(A'\cap B)=0.2$ then find $P(A'\cup B')$ **(1)**

(D) (1) State the difference between RAM and ROM **(1)**

(2) Convert $(1101.011)_2$ into decimal and octal form. **(2)**

Q.10 (A) (1)(i) State the axiomatic definition of probability. **(1)**

(ii) For events A and B prove that $P(A\cup B)\le P(A)+P(B)$ **(1)**

OR

(1) Prove : If A and B are events then if $A\subset B$ then $P(B-A)=P(B)-P(A)$ and $P(A)\le P(B)$ **(2)**

(2) State and prove the rule of baye's for two events. **(2)**

(B) Calculate any two. **(4)**

(1) Two cards are drawn from a pack of 52 cards. What is the probability that either both are red or both are kings ?

(2) If A and B are independent events associated with a random experiment then show that (i) A', B (ii) A', B' are also independent events.

(3) A factory has two machines A and B part records show that the machine A produced 60% of the items of output and machine B produced 40% of the items. Further 2% of the items produced by machine A were defective and 1 % produced by machine B were defective. It an item is drown at random, what is the probability that at is defective ?

(C) (1) A word consists of 9 letters; 5 consonants and 4 vowels. Three letter are chosen at random. What is the probability that more than one vowel is selected ? **(2)**

(2) (i) If $P(A')=0.7,\ P(B)=0.7$ and $P\left(B/A\right)=0.5$ then find $P(A\cup B)$ **(1)**

(ii) A problem in mathematics is given to 3 students whose chances of solving it are $\frac{1}{2},\frac{1}{3},\frac{1}{4}$ what is the probability that the problem is solved ? **(1)**

(D) (1) How many types of arithmetic are available in a computer ? Name them. **(1)**

(2) Convert $(101100.0011)_2$ into octal form.

Q.11 (A) (1)(i) For impossible event $\varnothing$ prove that $P(\varnothing)=0$ **(1)**

(ii) For every event A prove that $0\le P(A)\le 1$ **(1)**

OR

(1) Define (i) Equiprobable Primary events. **(1)**

(ii) Intersection events.

(2) If A and B are independent events then prove that A and B', A' and B and A' and B' are also independent events. **(2)**

(B) Calculate any two. **(4)**

(1) The percentage of students passing in three subjects A B and C are show below A : 50%, B : 40%, C : 30%, A and B : 35% B and C : 20%, A and C : 25%, A and B and C : 15%, Find the percentage of students passing in at least one subject.

(2) If A_1, A_2 and A_3 one mutualy independent events then prove that A_1', $A_2 \cap A_3'$ are independent events.

(3) If for the events $P(A)=P(B)=\frac{1}{4}\ P(A \cup B)=2/3$ then find $P(A' \cap B)$ and $P(A' \cup B)$

(C) (1) A card is drawn at randow from a pack to well shuffled 52 cards. If A and B denote the events that the card draw in black colour and the card drawn is a face card respectively then find $P(A \cup B)$ **(2)**

(2) Fill in the blanks by choosing the proper alternative (with necessary calculations) from those given in the brackets. **(2)**

(1) The probability of setting 100 marks in the question paper of board which is in your hand is .... (0, $3/4$, 1 not given)

(2) On tossing 3 coins once the probability of getting 2 consicative T is .... $\left(\frac{1}{4},\frac{1}{2},\frac{3}{8},\frac{3}{4}\right)$

(D) (i) State the names of electrinic components used in the third and forth generation of computers. **(1)**

(ii) Substract $.4891 \in -07$ from $.1645E-05$ **(2)**